# PINCH ANALYSIS

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## Summary

**Pinch analysis** is a technique that has been developed to identify the possible energy recovery by counter-current heat exchange between the hot streams to be cooled down and the cold streams to be heated up in a system. The pinch analysis is based on the definition of the **minimum approach temperature** $(\Delta T_{\min})$ that represents the energy-capital trade-off between the energy savings obtained by heat exchange and the required heat exchangers investment. For a given system, a pinch analysis is made in three steps: 1) the definition hot and cold streams, 2) the calculation of the minimum energy requirement (targeting step) and 3) the design of the heat exchanger network (**synthesis step**). The first step relies on the definition of the process unit operations and their required thermodynamic operating conditions. The second step of the analysis is made by computing the hot and cold composite curves of the process and identifying the **pinch point** location. The **hot and cold composite curves** represent respectively as a function of the temperature, the heat load available for heat exchange in the hot streams of the process and the heat required by the cold streams. The pinch point is identified by computing the heat cascade of the process that represents the maximum heat recovery between the hot and the cold streams, considering the minimum approach temperature constraint. The identification of the pinch point allows one to compute the **minimum energy requirement** of the process prior to any heat exchangers network rearrangement. The method allows one to target the possible energy savings and to identify what goes wrong with the present heat exchange system in the process. Based on the pinchpoint location, **heat exchanger networks** that realize the targeted heat recovery can then be synthesized using either heuristic and feasibility rules or applying mathematical programming methods. The pinch analysis method has been applied in various industrial sectors where heat transfer plays an important role. By its holistic nature, pinch analysis allows one to solve large scale problems like industrial sites or eco-industrial clusters.



## 1. Introduction

Pinch analysis is a method that aims at identifying the heat recovery opportunities by heat exchange in complex thermal processes. Based on the pioneering work of Umeda, the pinch analysis has been mainly developed in the early 70’s by Linnhoff and co-workers who developed a graphical method to calculate the minimum energy requirement of a process and design the heat recovery exchanger network and by Grossmann and co-workers who developed a mathematical programming framework for the design of heat exchanger networks. The graphical tools and the mathematical methods have converged to propose nowadays tools and methods that help in the identification of energy recovery by heat exchange and energy savings in site wide complex systems. The pinch analysis has been first developed for studying energy savings in the chemical process industry. Since then, the pinch analysis has been applied in the other industrial sectors where thermal operations occur like food, cement, pulp and paper, metallurgy, power plants, urban systems, etc.

The power of the pinch analysis stands mainly in its ability to offer a holistic analysis of the possible heat exchanges in a large and integrated system.

## 2. Energy-Capital Trade-off for Heat Recovery by a Heat Exchanger.

When analyzing a process system, the basic goal of the pinch analysis is to identify the possible heat recovery between the streams to be cooled down (so-called **hot streams**) and the streams to be heated up (so-called the **cold streams**) by using counter-current heat exchangers.

Let us consider the case of one cold stream to be heated up from an initial temperature $\left(T_{\text{cold,in}}\right)$ to a target temperature $\left(T_{\text{cold,target}}\right)$ and one hot stream to be cooled down from $T_{\text{hot,in}}$ to $T_{\text{hot,target}}$ (Figure 1). Assuming a constant specific heat capacity $\left(c_{\text{p cold}}\right)$, the heat required by the cold stream is calculated by $\dot{Q}_{\text{cold}} = \dot{M}_{\text{cold}} c_{\text{p cold}}(T_{\text{cold,target}} - T_{\text{cold,in}})$ and the heat available in the hot stream is $\dot{Q}_{\text{hot}} = \dot{M}_{\text{hot}} c_{\text{p hot}}(T_{\text{hot,in}} - T_{\text{hot,target}})$. Without heat recovery, the annual cost of the energy requirement $\left(OC^{ref}\right)$ is computed by Eq. (1) considering $c^{+}$, the cost of the hot utility used to supply the heat to the hot stream, $c^{-}$, the cost of the cold utility used to cool down the hot stream and $time_{\text{year}}$, the annual operating time of the process.

$$OC^{\text{ref}} = (c^{+}\dot{Q}_{\text{hot}} + c^{-}\dot{Q}_{\text{cold}}) \cdot time_{\text{year}} \quad (1)$$

A counter-current heat exchanger may be used to recover the heat from the hot stream in order to preheat the cold stream (Figure 1). The energy savings correspond to the heat load exchanged in the heat exchanger $\left(\dot{Q}_{\text{ex}}\right)$, it is obtained at the expense of an investment $\left(I_{\text{ex}}\right)$ that is a function of the heat exchanger area $\left(A_{\text{ex}}\right)$. The heat exchange area is computed from Eq. (2).

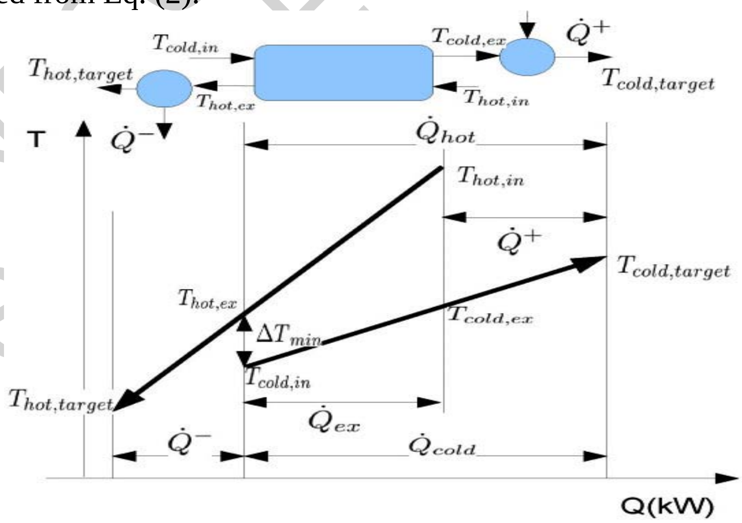


Figure 1. Heat exchanger example

$$\dot{Q}_{\text{ex}} = \dot{M}_{\text{hot}} c_{\text{p hot}}(T_{\text{hot,in}} - T_{\text{hot,ex}}) = \dot{M}_{\text{cold}} c_{\text{p cold}}(T_{\text{cold,ex}} - T_{\text{cold,in}})$$

$$\dot{Q}_{ex} = U_{ex} A_{ex} \frac{\left(T_{hot,in} - T_{cold,ex}\right) - \left(T_{hot,ex} - T_{cold,in}\right)}{ln\left(\frac{\left(T_{hot,in} - T_{cold,ex}\right)}{\left(T_{hot,ex} - T_{cold,in}\right)}\right)} \quad (2)$$

$$\frac{1}{U_{ex}} = \frac{1}{\alpha_{cold}} + \frac{e}{\lambda} + \frac{1}{\alpha_{hot}}$$

With:

$U_{ex}\left[\text{kW/m}^2\text{/K}\right]$ the overall heat transfer coefficient of the heat exchanger;
$\alpha_{cold}\left[\text{kW/m}^2\text{/K}\right]$ the convective heat transfer coefficient of the cold stream;
$\alpha_{hot}\left[\text{kW/m}^2\text{/K}\right]$ the convective heat transfer coefficient of the hot stream;
$\lambda\left[\text{kW/m/K}\right]$ the thermal conductivity of the tubes;
$e\left[\text{m}\right]$ the thickness of the tubes.

The installed cost of the heat exchanger may for example be estimated by a power law relation: $I_{ex} = a_{ex}(A_{ex})^{b_{ex}}$. In order to compare the annual investment cost $IC_{ex}$ with the energy savings, an annualized value of the investment is obtained by considering the annualization interest rate $(i)$ and the expected lifetime of the equipment $(ny_{ex})$. The possible heat exchange is limited by the **approach temperature** between the hot and the cold stream in the heat exchanger. When the approach temperature is small, the energy savings are high but the investment required is also high, when the approach temperature is bigger, the investment decreases while the operating costs are increasing. The **minimum approach temperature** $(\Delta T_{min})$ is the smallest temperature difference between the hot and the cold streams in the heat exchanger. The minimum approach temperature can be used as a parameter to determine the optimal size of the heat exchanger. The calculation of the optimal value of the $\Delta T_{min}$ is shown on Figure 2. The trade-off curve is obtained by combining the annual operating cost $OC_{ex}(\Delta T_{min})$ considered over a yearly operating time of $time_{year}$ (Eq. (3)) and the annualized investment $IC_{ex}(\Delta T_{min})$ computed by Eq. (4).

$$\begin{aligned} OC_{ex}(\Delta T_{min}) = (c^{+}(\dot{Q}_{cold} - \dot{Q}_{ex}(\Delta T_{min})) \\ + c^{-}(\dot{Q}_{hot} - \dot{Q}_{ex}(\Delta T_{min}))) \cdot time_{year} \end{aligned} \quad (3)$$

The hot and cold streams heat load being constant, the energy saving of the hot utility is identical to the energy saving of the cold utility.

$$IC_{ex}(\Delta T_{min}) = (\frac{i(1+i)^{ny_{ex}}}{(1+i)^{ny_{ex}} - 1}) a_{ex}(A_{ex}(\Delta T_{min}))^{b_{ex}} \quad (4)$$

where

$$A_{ex}(\Delta T_{min}) = \frac{(1-\kappa)}{\dot{M}_{hot} c_{p\,hot} U_{ex}} (\ln((T_{hot,in} - T_{cold,in})(1-\kappa) + \kappa \Delta T_{min}) - \ln(\Delta T_{min}))$$

$$\kappa = \frac{\dot{M}_{hot} c_{p\,hot}}{\dot{M}_{cold} c_{p\,cold}}$$

$$\dot{Q}_{ex}(\Delta T_{min}) = \dot{M}_{hot} c_{p\,hot} (T_{hot,in} - (T_{cold,in} + \Delta T_{min})) \quad (5)$$

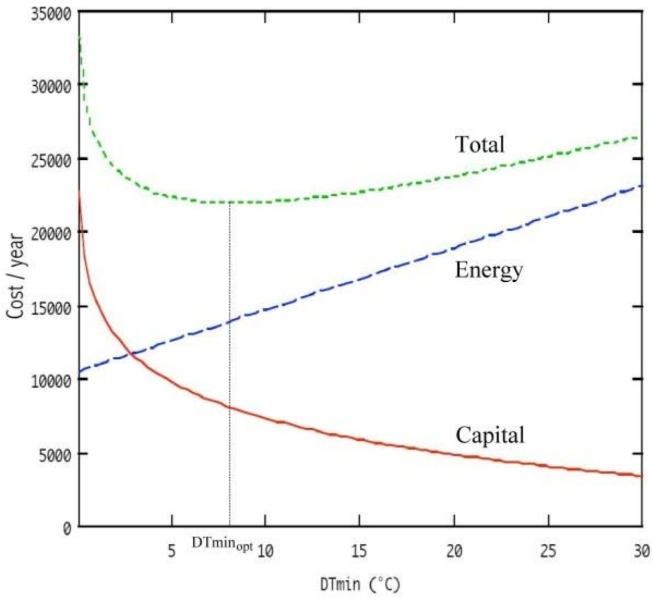


Figure 2. Energy Capital trade-off

## 3. Defining the Minimum Energy Requirement of a Process

### 3.1. The Composite Curves

When seeking to identify the possible heat recovery by heat exchange in a process, one has first to define the hot and cold streams. For this purpose, the process will be defined using the **energy flow diagram**. This diagram is obtained by considering the production process as a succession of unit operations that transform given inlet streams with specified thermodynamic states (temperature, pressure, composition and flowrate) conditions into resulting streams defined by their corresponding thermodynamic states. The hot and cold streams of the system will be defined by the necessary change of enthalpy (eventually pressure) between an **initial state** (outlet conditions of one unit operation or inlet state in the system) and a **target state** (required inlet conditions at the entry of the next operation or at the outlet of the system). From this definition, the process heat transfer requirement will be defined as a list of hot and cold streams.

Considering that all the exchange between the hot and the cold streams may be realized, the heat of the hot streams available for heat exchange in the process will be drawn as a function of the temperature. The obtained enthalpy-temperature profile will constitute the **hot composite curve**. The same approach is used for establishing the **cold composite curve** that represents as a function of the temperature the heat required from a heat exchange by the cold streams of the process.

The construction mechanism of the hot composite curve is illustrated for three hot streams in Figure 3 varying from $T_1$ to $T_5$. For each elementary temperature interval in the temperature range, the cumulated heat load is calculated as a sum of the contributions of each of the streams present. When using constant $c_\mathrm{p}$, the curve can be calculated by dividing the temperature range in successive linear segments defined by the extreme temperatures of the streams. For this construction, we assume that the streams have constant $c_\mathrm{p}$. Fluid phase changing streams or highly non linear $c_\mathrm{p}$ are represented into successive segments with constant $soP$. The heat load required is computed by $\sum_{h\in\{\text{Hot streams in interval k}\}} \dot{M}_h c_{\mathrm{p}\,h,k}(T_k - T_{k-1})$.

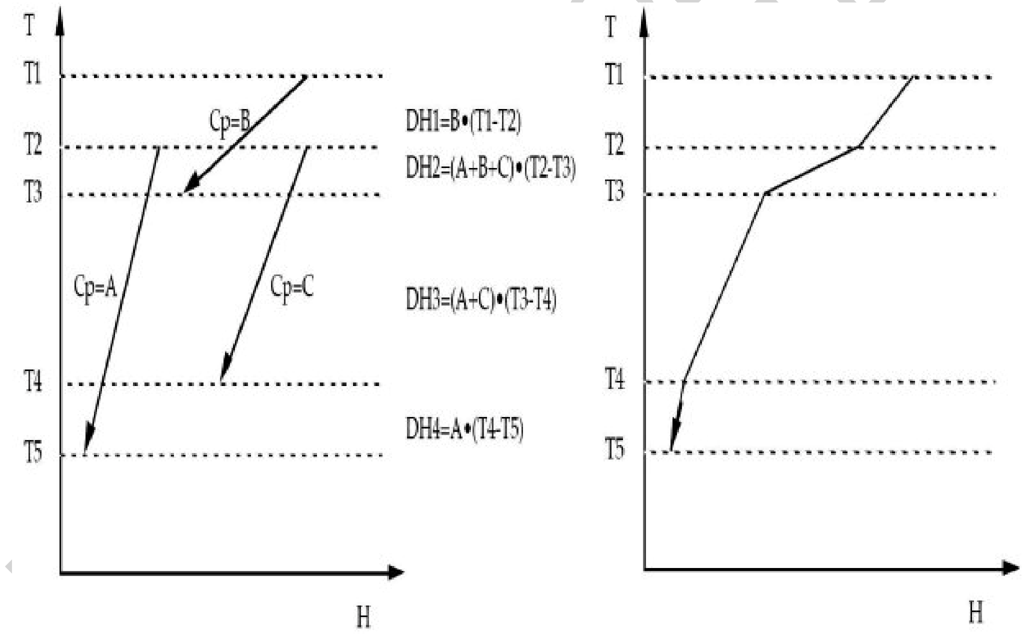


Figure 3. Hot composite curve construction

For the whole system, the composite curves can be seen as hot and cold streams that could exchange heat using counter-current heat exchangers (Figure 4). Heat recovery between hot and cold composite curves is feasible when the hot composite is above the cold composite. Assuming a $\Delta T_{\min}$ value, the cold composite may be shifted horizontally until the smallest vertical distance between the two composite reaches the $\Delta T_{\min}$ value. Like in the two streams example we can then read on the figure the minimum hot $\left(\dot{Q}^+\right)$ and cold $\left(\dot{Q}^-\right)$ energy requirement.

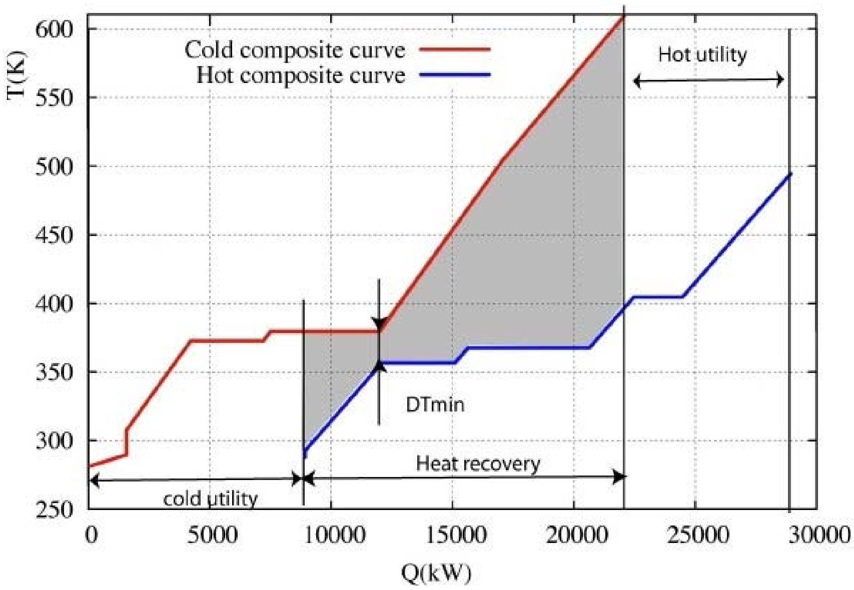


Figure 4. Hot and cold composite curves of a process

### 3.2. The Pinch Point

The point where the approach temperature between the two curves is equal to the chosen $\Delta T_{\min}$ value is called the **pinch point**. Usually, the pinch point does not appear at an extreme temperature like in the two streams exchange. Its position localizes the temperature of the process where the heat transfer is the most difficult and the temperature differences (the driving force) in the heat exchangers will be the smallest. Away from this point, the exchanges will be easier with higher approach temperatures. The pinch point identifies the bottleneck of the process in terms of heat recovery potential. The analysis of the streams in the vicinity of the pinch point will be of great help to further improve the energy efficiency of the process by changing the operating conditions of the unit operations concerned in order to create new energy recovery opportunities.

### 3.3. The Heat Cascade

Mathematically, the minimum energy requirement is computed by solving the heat cascade (8). This model is based on the definition of the corrected temperatures that are obtained by reducing the initial and target temperatures of the hot streams by $\frac{\Delta T_{\min}}{2}$ and increasing the temperatures of the cold streams by $\frac{\Delta T_{\min}}{2}$ (Eq. (6))

$$T_h^* = T_h - \frac{\Delta T_{\min}}{2} \; \forall h \in \{hot\ streams\} \quad (6)$$

$$T_c^* = T_c + \frac{\Delta T_{\min}}{2} \; \forall c \in \{cold\ streams\} \quad (7)$$

When a pinch occurs between the hot stream $h$ and the cold stream $c$, the approach temperature between the two streams is equal to $\Delta T_{\min}$. When $T_h - T_c = \Delta T_{\min}$, then $T_h^* - T_c^* = 0$ which corresponds to an intersection between the two curves.

The corrected temperatures define an ordered list of $n_r + 1$ increasing temperatures. A temperature interval $r$ is defined by two successive temperatures: from $T_r^*$ to $T_{r+1}^*$. Considering $R_r$, the heat cascaded from the system at a temperature higher than $T_r$, the energy balance may be written for each temperature interval. The heat cascade model (Eq. (8)) is a one degree of freedom linear programming problem that computes the minimum energy required $\dot{Q}^+ = R_{n_r+1}$ to balance the needs of the cold streams when recovering the maximum energy from the hot streams by counter-current heat exchange and cascading the heat excess to the lower temperatures.

$$\min_{R_r} \dot{Q}^+ = R_{n_r+1} \qquad (8)$$

subject to heat balance of the temperature intervals:

$$+ \sum_{h_r \in \{\text{hot streams in interval } r\}} \dot{M}_{h_r} c_{\mathrm{p}\,h_r} (T_{r+1}^* - T_r^*) \qquad (9)$$

$$- \sum_{c_r \in \{\text{cold streams in interval } r\}} \dot{M}_{c_r} c_{\mathrm{p}\,c_r} (T_{r+1}^* - T_r^*) \forall r = 1, \ldots, n_r$$

and the heat cascade feasibility

$$R_r \geq 0 \forall r = 1, \ldots, n_r + 1 \qquad (10)$$

With this definition, the value of the heat cascaded from the highest temperature $\left(R_{n_r+1}\right)$ represents the *minimum energy requirement* (**MER**) of the process $\left(\dot{Q}^+\right)$. It is assumed to be supplied to the process with a hot utility stream with a temperature higher than $T_{n_r+1}^* + \frac{\Delta T_{\min}}{2}$. By heat balance, $\dot{Q}^- = R_1$ represents the heat to be removed from the process by a cold utility with an expected temperature lower than $T_1^* - \frac{\Delta T_{\min}}{2}$.

The corrected temperature $T_{r_{\min}}^*$ corresponding to the inequality constraint $R_{r_{\min}} = 0$ is the pinch point temperature, it corresponds to a real temperature of $T_{r_{\min}}^* + \frac{\Delta T_{\min}}{2}$ for the hot streams and $T_{r_{\min}}^* - \frac{\Delta T_{\min}}{2}$ for the cold streams. When $r_{\min} = 1$ or $r_{\min} = n_r + 1$ the problem is said to be a *threshold* problem with respectively no cold utility or hot utility and without pinch point.

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**Biographical Sketch**

**François M. A. Marechal** is a chemical engineer (1986) from the University of Liège (B) and got his PhD degree in 1995 in the field of the energy analysis and synthesis of industrial processes. From 1986 to 2001, he worked as a researcher in the Prof. B. Kalitventzeff's group in the University of Liège. He joined the Industrial Energy Systems Laboratory (Prof D. Favrat) in Ecole Polytechnique Fédérale de Lausanne (EPFL) in 2001 where he is leading the computer aided energy systems analysis and design group. He teaches process integration, process modeling and optimization in the School of Mechanical Engineering. He authored more than 50 scientific papers in the field of computer aided process system engineering and process integration. His major research interests are rational use of energy in the industry, large scale process integration and thermo-economic optimal design of energy conversion systems using multi-objective optimization techniques.