NEWTONIAN AND NON-NEWTONIAN FLOW

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Keywords: Beverages, chocolate, Hooke's law, ketchup, Newtonian flow, rheology, rheopexy, thixotropy, viscoelasticity, viscometer, viscosity, yield stress

Contents

Summary

Rheology is the study of how fluids flow and solids deform when subjected to forces. While some foods can be classified as ideal solids (Hooke's Law) or ideal fluids (Newtonian), most behave as Non-Newtonian fluids or even exhibit viscoelastic behavior, i.e., their response can be expressed in terms of a viscous and an elastic component. These properties make the rheological characterization of foods a challenging task. Adding complexity, many foods (e.g., ketchup) exhibit time-dependent characteristics, i.e., their rheological behavior will depend on the previous history of the forces applied to the food. Furthermore, their response is greatly affected by the temperature and the concentration of components, and some foods (e.g., mayonnaise and yogurt) exhibit yield stress, i.e., they act as solids below a given stress, after which they flow. This article describes the rheological properties of Newtonian and non-Newtonian fluid foods, discusses techniques to measure their rheological properties, and gives examples of typical foods, such as chocolate, beverages, and ketchup.

1. Introduction

Rheology is a science that studies the flow and deformation of solids and fluids under the influence of mechanical forces. In order to study the rheological behavior of many products, it is necessary to resort to rheometry, which is utilized in different fields of the industry. The rheological measure of a product during the factory stage could serve as a control of quality for the said product. It could also correlate the microstructure of a product with its rheological behavior, which permits the development of new materials. This rheometry results in rheological equations that assist in the engineering of processes, above all in the unit operations involving the transfer of heat and momentum. Finally, by knowing the demands of the consumer, it is possible to develop an adequate product to meet such demands.

Many industries frequently work with products that are in a liquid phase during all or some of the industrial operations carried out (concentration, evaporation, pasteurization, pumps, and those in between), thus a good design of each process installation is indispensable for good operation. In the design of all processes, it is necessary to know the individual physical characteristics differentiating each process. One characteristic is the rheological behavior of the fluid processed. Knowledge of its rheology can avoid possible excess dimensions of pumps, pipes, evaporators, etc., all of which could cause a negative rebound in the economy of the process.

The viscosity is utilized in calculating the parameters of transport phenomena of momentum and energy, as well as for the quality control of certain products. As a result, in the equations of the mathematical model expounded for the diverse operations that form a particular process, the rheological constants of the fluids should be determined by means of experimentation. For this reason, it is very important that the rheological characterization of the several flowing currents and the deduction of equations be done in order to be able to calculate directly the rheological constants as a function of the considered food and of the variables of the operation.

2. Stress and Deformation

When a force is applied to a body, different response patterns occur depending on the material. For instance, an elastic solid is deformed when subjected to a force. However, when the force is removed, the solid will recover its initial shape. If the material is a Newtonian fluid, it will flow when subjected to a force and will gradually stop flowing when the force is removed, but will not recover its original shape.

The stress (σ) applied to a material is defined as the force (F) per unit of area (A). In this way, the stress can be normal and/or tangential. There are two types of normal stress, traction and compression stress. Nine components of the stress exist for a body (Figure 1). Each one is designated with two subscripts "i" and "j", expressing the section that receives a given stress and the direction of the stress, respectively. So, σ_{ij} is a stress applied on the *i*th section in the *j*th direction. The nine components of the stress form the stress tensor:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{22} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \qquad (1)$$

The stress tensor is symmetric, i.e. $\sigma_{ij} = \sigma_{ji}$. Therefore, only six components of tensor are independent.

For shear stress $i \neq j$, while for normal stresses $i = j$. The normal stress of traction is defined as positive $(\sigma_{ii} > 0)$, while the normal stress for compression as negative $(\sigma_{ii} < 0)$.

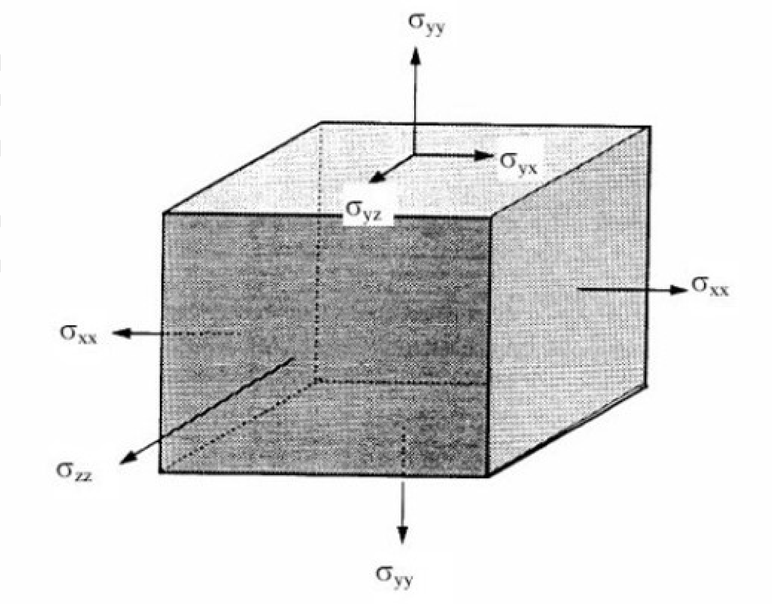

Figure 1. Stress applied on a material.

All stress applied to a material produces a deformation. The deformation can be angular or longitudinal, depending on the direction of the applied stress. Normal stress produces longitudinal deformation (extensions or reduction), while shear stress produces angular deformation.

If normal traction stress is applied to a bar of length L_0 (Figure 2), it will produce an extension in such a form that the final length of the bar will be $L = L_0 + \Delta L$. The deformation produced on this bar is expressed as the Cauchy strain $\left(\varepsilon_c\right)$:

$$\varepsilon_c = \frac{\Delta L}{L_0} = \frac{L}{L_0} - 1 \tag{2}$$

For high deformations, it is preferable to use the Hencky strain (ε_H), defined as:

$$\varepsilon_H = \ln\left(L/L_0\right) \tag{3}$$

When shear stress is applied, it produces a deformation as shown in Figure 3, where the inferior face of the body remains stationary. The angle of shear is calculated according to the equation:

$$\tan\gamma = \frac{\Delta L}{L} \tag{4}$$

For small deformations, where $\tan \gamma \cong \gamma$, the tangent of the angle of deformation is considered equal to the shear strain (γ).

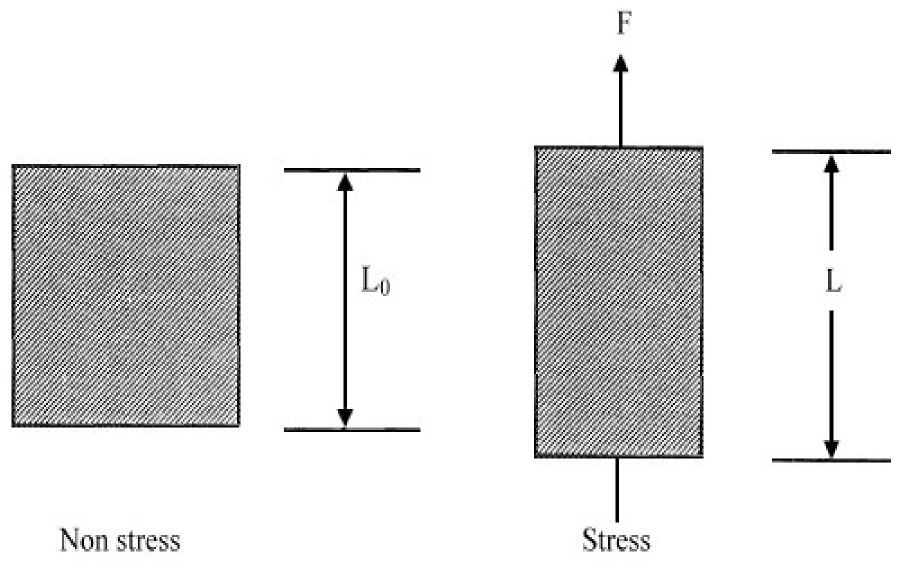

Figure 2. Deformation in normal stress.

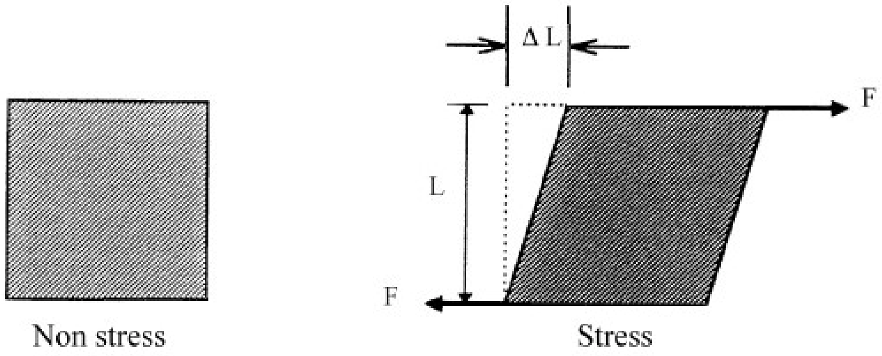

Figure 3. Deformation in shear stress.

3. Elastic Solids and Newtonian Fluids

A shear stress (σ_{ij}) applied to a solid material will produce a shear strain (γ), according to Equation 5:

$$\sigma_{ij} = G\gamma \qquad (5)$$

The proportional constant G is called shear modulus or rigidity modulus and the material is denominated Hookean solid, being linearity elastic and exhibiting no flow. The stress remains constant until the deformation is eliminated, returning the material to its initial shape.

If a normal stress (σ_{ii}) is applied to a Hookean solid, the stress will be proportional to the Cauchy strain:

$$\sigma_{ii} = E\varepsilon_C \qquad (6)$$

where the proportional constant E is denominated as the Young modulus, or elasticity modulus.

Elastic solids are those deformed when subjected to shear or normal forces, but which recover their initial form if the applied force is removed. Solids that do not recover their initial shape once an applied force has stopped are known as solid plastics. Elastoplastic solids exhibit intermediate behavior, i.e., under little stress they behave as solids, but once a critical stress is reached they will not recover their initial shape.

Fluids, i.e., materials that exhibit flow when subjected to stress, are usually contained between two parallel plates of gap *h* (Figure 4), to study their rheological characteristics. The inferior plate remains static, while the superior one moves at constant speed (v_P), as shown in Equation 7:

$$v_P = \frac{dx}{dt} \qquad (7)$$

where x is displacement and t is time.

In order to maintain a constant speed, a shear stress (σ_{12}) needs to be applied to the upper plate while the lower plate remains static. In this way, a profile of speeds occurs in the fluid along the gap between the two plates (Figure 4) and the fluid will flow at a rate of deformation or shear strain rate ($\dot{\gamma}$), defined as:

$$\dot{\gamma} = \frac{dv_P}{dy} = \frac{d}{dy}\left(\frac{\delta x}{\delta t}\right) = \frac{d}{dt}\left(\frac{\delta x}{\delta y}\right) = \frac{d\gamma}{dt} \tag{8}$$

The shear stress applied σ_{12} and the shear strain rate $(\dot{\gamma})$ are directly proportional:

$$\sigma_{12} = \eta\dot{\gamma} \tag{9}$$

This expression is known as Newton's law of viscosity, and the constant of proportionality η is the Newtonian viscosity.

Fluids that follow Newton's law of viscosity are considered ideal or Newtonian fluids.

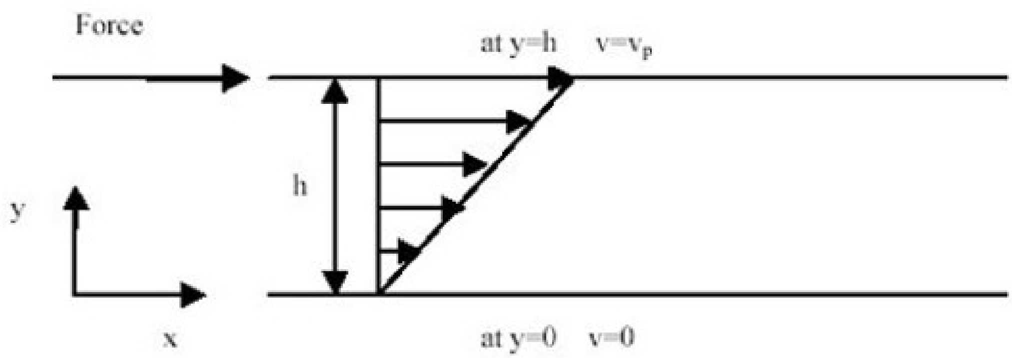

Figure 4. Velocity profile between parallel plates.

4. Viscometric Functions

For a complete characterization of fluids, a study of stresses and deformations in the three-dimensional space is required. For a flow in simple shear, the stress tensor is reduced:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \tag{10}$$

Simple shear flow is also called viscometric flow, and is used to characterize the behavior of fluids under geometries such as concentric cylinders, axial flow in a tube, and rotational flow between plate-plate and cone-plate surfaces.

In simple shear flow, only three functions are required to describe the behavior of a fluid. The so-called viscometric functions are the viscosity function (11) and the first (12) and second (13) normal stress coefficient functions, defined as:

$$\eta(\dot{\gamma}) = \phi_1(\dot{\gamma}) = \frac{\sigma_{12}}{\dot{\gamma}} \qquad (11)$$

$$\psi_1(\dot{\gamma}) = \phi_2(\dot{\gamma}) = \frac{\sigma_{11} - \sigma_{22}}{\dot{\gamma}^2} \qquad (12)$$

$$\psi_2(\dot{\gamma}) = \phi_3(\dot{\gamma}) = \frac{\sigma_{22} - \sigma_{33}}{\dot{\gamma}^2} \qquad (13)$$

Equations (12) and (13) are sometimes expressed in terms of the first and second normal stress difference:

$$N_1 = \sigma_{11} - \sigma_{22} \qquad (14)$$

$$N_2 = \sigma_{22} - \sigma_{33} \qquad (15)$$

Since N_2 is difficult to measure and usually very low, it is assumed null in most cases.

5. Rheological Classification of Fluids

In classical mechanics, the distinction between liquids and solids was very clear, and separate physical laws existed to describe the behavior of solids (Hooke’s law) and liquids (Newton’s law). However, a variety of products (such as foods) exist that exhibit intermediate behavior which needs to be well characterized.

Fluids are initially classified as having Newtonian or Non-Newtonian behavior, depending on whether they can be described by Newton's law of viscosity or not. Non-Newtonian fluids are also classified as time-dependent or time-independent. Fluids in which rheological behavior depends only on the shear stress (at constant temperature) are considered time-independent. Time-dependent fluids are those in which the viscosity depends, not only on the shear stress, but also on the amount of time the stress has been applied to the fluid. There are fluids that present both viscous and elastic behavior; they are called viscoelastic fluids.

Classification of flowing fluids can be done by means of viscometric functions, defined in the previous section. For Newtonian fluids, the viscosity function is constant, and the viscosity (Newtonian viscosity) is independent of shear strain rate and time ($\eta(\dot{\gamma}) = \eta =$ constant).

In non-Newtonian fluids, the viscosity function depends on the shear strain rate, and the apparent viscosity is defined as:

$$\eta_a = \frac{\sigma_{12}}{\dot{\gamma}} = \eta(\dot{\gamma}) \tag{16}$$

In this way, fluids can be classified according to the following scheme:

A). Newtonian flow
B). Non-Newtonian flow
- 1). Time independent
  - a) Plastic fluids
  - b) Pseudoplastic fluids (shear-thinning)
  - c) Dilatent fluids (shear-thickening)
- 2). Time dependent
  - a) Thixotropic fluids
  - b) Antithixotropic or rheopectic fluids

C). Viscoelastic fluids

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Biographical Sketches

Albert Ibarz received his B.S. and Ph.D. in Chemical Engineering at the University of Barcelona (Spain) and Technical Engineering in Food Industries at Polytechnic University of Catalunya at Barcelona (Spain). He is currently Professor of Food Engineering at the University of Lleida (Spain) and is actually Vice-Chancellor at the same university. Dr. Albert Ibarz chaired the Organizing Committee for the 2002 Spanish Congress of Food Engineering, and has authored more than 100 research papers, as well as four books and several book chapters on Food Engineering.

M. Elena Castell-Pérez is a native of Venezuela. She earned a Bachelor of Science in Food Engineering from Campinas State University in 1980, and returned to Venezuela where she worked as Chief Engineer of a yogurt production plant. She received a Master of Science and Doctor of Philosophy from the Department of Agricultural Engineering at Michigan State University in 1984 and 1990, respectively. Her dissertation research focused on the fundamental evaluation of mixer viscometry for rheological characterization of non-Newtonian fluid foods. This work was motivated by the need for new instrumentation and methodologies for use in food process engineering. Dr. Castell-Pérez has been actively involved in research in the area of food rheology, engineering properties of materials, and alternative uses of food and agricultural materials such as films for packaging applications. She joined the faculty of the Department of Food and Animal Sciences at Alabama A&M University in 1991, where she developed a research and teaching program in Food Engineering. Currently, she is Professor at the Department of Biological and Agricultural Engineering at Texas A&M University where she teaches courses in material properties, packaging, and food plant unit operations. Dr. Castell-Pérez's current research emphasizes the characterization of biological materials to determine their value as engineering materials, as well as critical factors affecting their behavior during and after processing. She is a member of the American Society of Agricultural Engineers, Institute of Food Technology, and the Society of Rheology. Dr. Castell-Pérez has published 20 papers in professional journals, one book and two book chapters, and more than 30 abstracts and proceedings. She is a licensed Professional Engineer (Texas).

Gustavo V. Barbosa-Cánovas received his B.S. in Mechanical Engineering at the University of Uruguay and his M.S. and Ph.D. in Food Engineering at the University of Massachusetts-Amherst. He then worked as an Assistant Professor at the University of Puerto Rico from 1985-1990, during which he was granted two National Science Foundation (NSF) awards for research productivity. Following this, he went to Washington State University (WSU) where he is now Professor of Food Engineering and Director of the Center for Non-Thermal Processing of Food (CNPF). Dr. Barbosa-Cánovas chaired the Organizing Committee for the 1997 and 1999 Conference of Food Engineering (CoFE). In addition, he is Editor of the journal *Food Science and Technology International* published by SAGE, the journal *Innovative Food Science and Emerging Technologies* published by Elsevier Science, and the Food Engineering theme in the Encyclopedia of Life Support Systems (EOLSS) to be published by UNESCO. Dr. Barbosa-Cánovas is Editor-in-Chief of the Food Engineering Book Series published by Kluwert Academic and Plenum Publishers (KAPP) as well as of the Food Preservation Technology Book Series published by CRC Press. He has chaired and organized several technical sessions at the American Institute of Chemical Engineers (AIChE) and Institute of Food Technologists (IFT) annual meetings, edited 12 books on Food Engineering topics, and authored, among others, *Dehydration of Foods* (Chapman and Hall), *Nonthermal Preservation of Foods* (Marcel Dekker), *Food Engineering Laboratory Manual* (Technomic), and

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