FROM GEOMETRY TO TOPOLOGY

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Contents

1. Introduction
2. Three Visions of Geometry in the late Nineteenth Century
2.1. Projective Geometry as a Fundamental Geometry
2.2. Non-Euclidean Geometry and Physical Space
2.3. The Kleinian View of Geometry
2.4. Geometry in Spaces of any Number of Dimensions
2.5. The Search for an Axiomatic Foundation of Geometry
3. Main Roots to Topology
3.1. The Classification of Surfaces
3.2. Complex Function Theory and the Birth of Manifolds
3.3. Fourier Series and Topology of Point Sets
3.4. The Cantor Set
3.5. Construction of the Real Number System
3.6. The Dirichlet Problem and Topology of Planar Domains
4. Creating a Theory of Geometric Topology
4.1. The Alexander Horned Sphere
4.2. The Brouwer Fixed Point Theorem
4.3. Shaping the Notion of Dimension
4.4. Topology of Manifolds
5. Differential Geometry and Topology
5.1. Vector Fields on Manifolds
5.2. Curvature of Curves and Surfaces
5.3. Theorema Egregium
5.4. Surfaces of Constant Curvature and the Uniformization Theorem
5.5. The Gauss-Bonnet Theorem and Generalizations
6. Creating a Theory of Point-set Topology
6.1. The Beginnings to Point-set Topology
6.2. Point-set Topology and the Calculus of Variations
6.3. The General Notion of a Topological Space
7. Concluding Remarks
7.1. Algebraic Topology of Manifolds
7.2. Point-set Topology and Fractal Structures
Glossary
Bibliography

Biographical Sketch

Summary

This chapter follows the developments in algebra and analysis that eventually led to the creation of a new branch of geometry with focus on the qualitative aspects of geometry, originally termed *analysis situs*, or *geometria situs*, by Leibniz, but since 1836 better known under the name *topology*, suggested by the German mathematician Listing.

1. Introduction

In a memoir of 1679, the German philosopher Gottfried Wilhelm Leibniz (1646–1716) set himself the goal of formulating some fundamental properties of geometrical figures, using special symbols to represent them, and combining these properties to make others. He called his studies *analysis situs*, or, *geometria situs*. It is slightly unclear what he meant, but in a letter to Huygens 1679 he explained that he was unsatisfied with the way coordinate geometry handles geometrical figures because it involves quantities. Leibniz searched for another form of analysis "which is truly geometrical by expressing position (*situs*) directly, in the same way as algebra expresses magnitude".

Leibniz did not immediately stimulate any new developments with his vague ideas, but in 1735, the Swiss mathematician Leonhard Euler (1707–1783) published an article with the title *Solution of a problem from geometria situs*. The article provides a solution to the problem known as the *Königsberger bridge problem*, and the title of the article clearly indicates that Euler considered it to be a contribution in the spirit of the ideas put forward by Leibniz. Today one would probably say that Euler misunderstood Leibniz's intentions by referring to his investigations as a contribution to *geometria situs*. The solution to the Königsberger bridge problem is nowadays counted as the first proper contribution to *graph theory*, although Euler makes no mention of the notion of a graph in either this or in any other of his papers.

Euler did, however, make a pioneering contribution in the field of *analysis situs*. His proof of the combinatorial property of the surface of a convex polyhedron, known as the *Euler's polyhedron formula*, in an article of 1750, clearly belongs to the study proposed by Leibniz. If you count the number V of vertices, the number E of edges, and the number F of faces on the surface of a convex polyhedron, then the alternating sum $V-E+F=2$. Euler's proof of this formula is now counted as the first proper contribution to *analysis situs*. The formula was certainly known by 1639 to the French philosopher and mathematician René Descartes (1596–1650) and through his unpublished manuscript also to Leibniz in 1675. (Presumably, the formula was known already to Archimedes (287–212 BC).).

In a letter dated April 1st, 1836, the German mathematician Johann Benedict Listing (1806–1882) suggested the name *topologie* for the study of geometrical figures following Leibniz. In print, the new name *topology* was used for the first time in the book *Vorstudien zur Topologie*, published by Listing in *Göttinger Studien* 1847. The word derived from '*topos*', the Greek expression for place or location, and 'logos', meaning words for, or the study of, something. The name *analysis situs* was still commonly used for qualitative studies of geometrical objects in the first half of the 20th century, but from about 1950, the name topology has been used exclusively.

In the late 19th and early 20th centuries topology gradually replaced geometry as the fundamental domain for large parts of mathematics. So we begin by looking at the dramatic state of research in geometry in the late 19th century, and we follow the developments that led first to the creation of a topology of manifolds, then to a renewed interest in axiomatic geometry. Then we look at the roots of topology. To this day topology has two identifiable overlapping parts: one often called *algebraic topology* and the other *point-set topology*. Algebraic topology has its roots in a geometric approach to complex analysis; point-set topology grew out of a variety of problems in real and complex analysis.

2. Three Visions of Geometry in the late Nineteenth Century

Geometry went through two major changes in the 19th century, or rather it experienced two major challenges, one largely successful and the other at first rebuffed. In 1800 the subject was largely confined to Euclidean geometry in two and three dimensions, often taught according to the pattern laid down in texts of *Euclid's Elements* (as it was in the influential text book *Éléments de Géométrie* by Legendre) or in the more algebraic style introduced in the 17th century by Descartes (Cartesian or coordinate geometry).

Gaspard Monge (1746–1818) in Paris was also interested in the projections of solid figures onto planes, for the use of engineers and architects, and in the 1810s and 1820s a number of French geometers, mostly his former students at the newly-founded École Polytechnique, promoted a study of the properties that geometric figures share with their shadows. The most important of these was Jean Victor Poncelet (1788–1867) whose *Traité des Propriétés projectives des figures* of 1822 showed how this new geometry, called *projective geometry*, could unify, simplify, and extend the study of conic sections.

2.1. Projective Geometry as a Fundamental Geometry

Contemporaries found some of Poncelet's methods unconvincing; however, another French geometer, Michel Chasles (1793–1880), showed how to replace them with a systematic use of the invariance of cross-ratio under projection. The *cross-ratio* of four points A, B, C, D on a line can be defined in various ways, all of them equivalent to $\frac{AB\cdot CD}{AD\cdot CB}$, and if these four points are projected (see the figure below) to the four points A', B', C', D' on another line then the cross-ratios are equal: $\frac{AB\cdot CD}{AD\cdot CB}=\frac{A'B'\cdot C'D'}{A'D'\cdot C'B'}$.

In some sense this property of four collinear points in projective geometry plays the role of distance, the separation between two points on a line, in Euclidean geometry. The same approach was also adopted independently by the Swiss mathematician Jakob Steiner (1796–1863), who taught projective geometry in Berlin, and projective geometry steadily became accepted as a fundamental new approach to geometry.

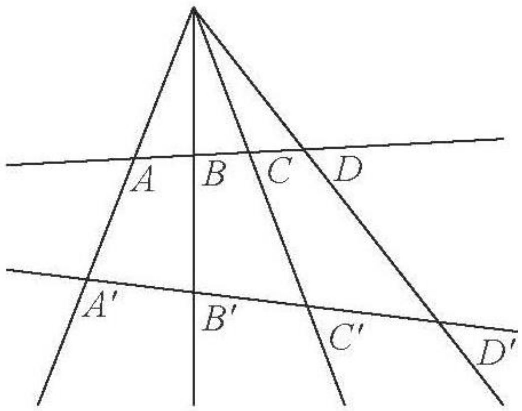

Figure 1. Cross-ratio under projection $\dfrac{AB \cdot CD}{AD \cdot CB} = \dfrac{A'B' \cdot C'D'}{A'D' \cdot C'B'}$

The properties a figure shares with its shadows are not many. It is easy to see that lengths are not shared – a shadow of a line segment can be many different lengths – and Poncelet called his new geometry non-metrical for that reason. This is quite a paradoxical name, since geometry, after all, means the measure of the earth, and geometry would seem to be about measured lengths. Neither do the angles in a figure agree with those in its shadows. But if a line meets a curve in three points their shadows meet in three points, and if a line touches a curve their shadows touch. The properties of intersection and tangency are shared; one says they are invariant under projection. And, as we have seen, there is a possibly unexpected property of four points on a line: they have a cross-ratio that is preserved under projection. A projective property is by definition one that is the same for a figure and any of its images under a projective transformation. Projective geometry is the study of the projective properties of figures and because projective properties are automatically true in Euclidean geometry it was gradually agreed by the middle of the 19th century that projective geometry is more fundamental than Euclidean geometry.

2.2. Non-Euclidean Geometry and Physical Space

At the same time, the late 1820s and early 1830s, Euclidean geometry was facing another, and arguably more fundamental challenge, but one that was held at bay for a further generation. This began as a concern about the nature of straight lines, in particular parallel lines (by definition, parallel lines are lines that never meet). The account of them in *Euclid's Elements* leads naturally to the idea that two parallel lines in a plane are everywhere the same distance apart. János Bolyai (1802–1860) in what is now Hungary, Nicolai Ivanovich Lobachevskii (1793–1856) in Kasan in Russia, and Carl Friedrich Gauss (1777–1855) in Göttingen, Germany, all investigated a geometry in which some pairs of straight lines might draw closer and then diverge and so never meet, and other pairs would draw closer and closer but still never meet, much as do the hyperbola and its asymptotes. However, Gauss, who was the dominant mathematician of his time, never published his ideas. Bolyai did, but only obscurely, and Lobachevskii, who did publish, found that his ideas were ignored. Had Gauss taken them up and lent his name to them, matters might have been different, but he gave them only half-hearted support, and it was left to mathematicians of the next generation (after Gauss, Bolyai and Lobachevskii were dead) to appreciate what they had done.

In particular, Bolyai and Lobachevskii had shown that a new geometry was possible, one that differed from Euclidean geometry in just one respect (the nature of parallel lines) but which had a perfectly good concept of length and angle and could in fact be the correct geometry of physical space. Indeed, they discovered a family of such geometries that depended on a parameter and had Euclidean geometry as a limiting case (when that parameter took the value 0).

Attempts to determine this parameter all failed, because it was soon clear that it must be very close to zero and too close to measure at the time, but the much more important point was that the new geometry was possible at all. It followed that Euclidean geometry could not be a priori true, as had hitherto been thought. It was surely this challenge to a deeply held belief that caused *non-Euclidean geometry* (as the new geometry came to be called) to be so strongly resisted.

2.3. The Kleinian View of Geometry

Matters changed when the German mathematician Friedrich Bernhard Riemann (1826–1866), who had briefly studied with Gauss in Göttingen, proposed that geometry is simply the study of a space of points with a concept of distance. In his habilitation lecture in Göttingen of 1854 with the title "*Über die Hypothesen, welche der Geometrie zugrunde liegen*", published only posthumously in 1867, Riemann showed how this program, which was a vast generalization of ideas due to Gauss in the 1820s, could be made to work. He indicated in passing how to vindicate non-Euclidean geometry, and he was followed by the Italian mathematician Eugenio Beltrami (1835–1900).

Both men argued that just as a description of the Earth in an atlas makes it possible to navigate on the (spherical) Earth and to do geometry on a sphere, so too there is a description in an "atlas", which they described explicitly in formulae, of non-Euclidean geometry. This new description eliminated doubts about non-Euclidean geometry in the minds of mathematicians (philosophers followed only slowly).

On the basis of Beltrami's "atlas", in which curves of shortest length in two-dimensional non-Euclidean geometry appear as straight chords inside a fixed circle, Felix Klein showed in the early 1870s how non-Euclidean geometry can be regarded as a special case of projective geometry. This gave him a way in which to proclaim that all the known geometries (Euclidean and non-Euclidean) are special cases of projective geometry; he did not know then about what is called affine geometry, but it fits in very naturally. Klein's unification of geometry became well-known when it was republished in the 1890s in several languages, by which time other mathematicians had made great strides in the study of geometry, and it is still called the Kleinian view of geometry.

It emphasizes that any geometry is the study of a space and a group of transformations that move figures around in that space without altering the fundamental properties of that geometry. If those properties are the projective ones the transformations are the projective transformations; if they preserve distance they are the metrical transformations of Euclidean or non-Euclidean geometry.

2.4. Geometry in Spaces of any Number of Dimensions

Although the Kleinian view of geometry did well, the Riemannian viewpoint is much more diverse and, arguably, more profound. It allows for geometries in spaces of any number of dimensions, and of a variety of shapes. One can, for example, study geometry on a torus, or on what is called a Riemann surface (the surface in four-dimensional space defined by an algebraic equation in two complex variables). In this connection in 1880 the young French mathematician Henri Poincaré (1854–1912) made the remarkable discovery that all but the simplest Riemann surfaces naturally acquire non-Euclidean geometry, thus showing that non-Euclidean geometry has a major role to play in the study of complex function theory. The study of spaces with geometry locally like Euclidean geometry but globally different (such as the torus) proceeded rapidly in the second half of the 19th century as a study of a variety of mathematical objects that can be given coordinates. This study remained, however, either a branch of pure mathematics or a tool in mechanics, and it was not seriously thought at this time that physical space could be other than Euclidean or non-Euclidean.

2.5. The Search for an Axiomatic Foundation of Geometry

One reasonable reaction to all these new geometries was to wonder what had gone wrong: how could all those textbooks modeled on *Euclid's Elements* have misled mathematicians and made them blind to the new possibilities? A critical re-examination of the arguments of such books, ancient and modern, led mathematicians to the view that such books were inherently flawed. Moritz Pasch (1843–1930), a German geometer, was among those who thought it would be better to start again and give projective geometry and Euclidean geometry new axiomatic foundations. He was followed by a number of mathematicians in Italy who gravitated around Giuseppe Peano (1858–1932), of whom Mario Pieri (1860–1913) was the most active.

They gave rigorous abstract axiomatic foundations for these geometries that, unlike Pasch, made no appeal to our beliefs about space and the objects in space. Their work proved less influential, however, than that of David Hilbert (1862–1943), the German mathematician who, with Poincaré, dominated mathematics at the start of the 20th Century. Hilbert also gave foundations of these geometries, but in so doing he successfully promoted the idea that the study of axiom systems was likely to be applicable across the whole of mathematics. This broad ambition, coupled with his powerful academic position and his brilliance in many other domains of mathematics, led to Hilbert's axiomatic geometries being the ones that are best remembered today.

3. Main Roots to Topology

By the end of the 19th century geometry was firmly associated with the idea of transformations of figures and the properties of figures that are unaltered by such transformations. Simultaneously but in other branches of mathematics transformations were being studied, and properties invoked, that were to prove much more fundamental.

These are associated with the name of topology. Topology has two main roots: one is algebraic and grew out of complex function theory; the other has to do with sets of points and grew up in both real and complex analysis.

3.1. The Classification of Surfaces

A key early success in the algebraic tradition was the classification of surfaces. The concept of a surface is an intuitive one, and the work of numerous mathematicians at the start of the 19th century led to the conclusion that surfaces can be distinguished by three characteristics. The first is obtained by drawing a net of curves on a surface that meet only at points (called vertices). This net must also divide the surface into disc-shaped regions (called faces) separated by the curves (called edges).

It turns out that however this is done, if you count the number V of vertices, the number E of edges, and the number F of faces on a given surface, then the alternating sum $V-E+F$ always gives the same value χ. The quantity $\chi = V-E+F$ is called the *Euler characteristic* of the surface. It is 2 for the sphere, 1 for the disc, 0 for the torus, and so on. A surface may also have a number of 'holes' or, somewhat more precisely, end in a number of distinct boundaries. This number is also used to classify surfaces. The third property is a little more elusive. It was to turn out that there are surfaces that do not permit one to define the notion of 'clockwise' turning in a coherent way along the surface.

The simplest of these is the Möbius band, which is obtained from a thin strip of paper that has been given a half-twist before one pair of opposite edges are glued together. Such a surface is said to be *non-orientable*; surfaces that permit one to define clockwise turning in a coherent way are called *orientable*.

It was to turn out that surfaces which have the same Euler characteristic, the same number of boundary components, and are either both orientable or both non-orientable are topologically equivalent. Two surfaces are topologically equivalent if each can be mapped onto the other in a one-to-one correspondence of points such that nearby points stay nearby (in more mathematical terminology, the maps between the surfaces are to be continuous).

The two mathematicians most responsible for this classification of surfaces worked independently. The German mathematician A.F. Möbius (1790–1868) published his account in 1865 (incidentally, it seems that J.B. Listing may have discovered it a few months before). He considered only those surfaces which can be embedded in three-dimensional Euclidean space (these are the surfaces that do not self-intersect). The treatment given by Camille Jordan (1838–1922), a French mathematician, in 1866 concentrated more on the different types of closed paths that can be drawn on a surface; in today's language that corresponds to looking at the homotopy groups of the surfaces, but although Jordan was on his way to becoming a leading figure in group theory he did not use that concept in these papers.

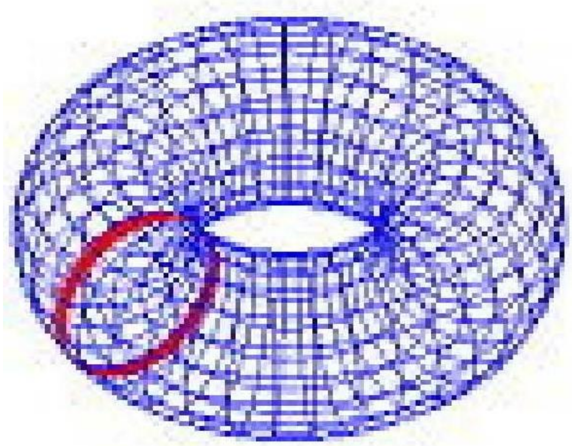

Figure 2. Torus with non-separating closed curve

3.2. Complex Function Theory and the Birth of Manifolds

Remarkably, ten years before this work was done in his ground breaking study of complex function theory and the integrals of algebraic functions, which redefined the theory of analytic functions in the 1850s, Riemann had shown that every algebraic function gives rise to an orientable surface with no boundary and that the properties of complex functions on the surface depend solely on the topological type of the surface. This made the classification of surfaces of immediate relevance to the study of major problems in complex function theory. His argument was that when one draws closed curves on an orientable surface one of two things happens: either the closed curve divides the surface into two pieces, as it necessarily does on the sphere or the disc, or it need not (this can happen on, for example, a torus).

Riemann's work invited generalization to higher dimensions, and this problem was taken up after Riemann's death in 1866 by his Italian friend Enrico Betti (1823–1892). In 1871 Betti began the study of k-dimensional subsets of Euclidean n-dimensional space. Following Riemann he tackled the problem by considering how these subsets can be chopped up into basic pieces by systems of $(k-1)$-dimensional cuts. But the great difficulties inherent in this work not only prevented Betti from getting very far, they also blocked progress for a long time. Decisive advances only came when Poincaré took up the subject in the 1890s. He had found it necessary to study higher-dimensional spaces in his work on celestial mechanics and in his study of Riemann surfaces, and thus motivated he spent much of the 1890s and early 1900s developing a constructive definition of manifolds which permitted him to deduce, or at least conjecture, many results. His work inspired others to join in and the first theorems on manifolds in dimensions greater than 2 date from this period.

However, on any orientable surface the process of drawing loops that do not divide the surface into two must stop, which it does when any new loop divides the surface, and when it does an odd number of loops, say $\gamma = 2p+1$, has been drawn (counting the first dividing loop). This number, which is 1 for a sphere, 3 for a torus, and so on, Riemann called *the order of connectivity* of the surface. If the surface is furthermore *closed*, i.e. it is of finite extent in space and has no boundary components, then the order of connectivity is related to the Euler characteristic of the orientable surface via the formula $\chi = 2 - 2p$. Möbius made no reference to Riemann's ideas in his paper, but Jordan was undoubtedly responding to Riemann's dissections of surfaces.

3.3. Fourier Series and Topology of Point Sets

At the same time, the study of real analysis was raising more and more delicate yet fundamental questions that directed attention to the behavior of sets of points on the real line or in the plane. Many of these questions arose in the study of Fourier series. The French mathematician Jean Baptiste Joseph Fourier (1768–1830) had claimed in 1822 that any function $f(x)$ with period 2π can be written as a series of sines and cosines in the form

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx), \qquad (1)$$

where $a_k = \int_{-\pi}^{\pi} f(x)\cos(kx)\,dx$ and $b_k = \int_{-\pi}^{\pi} f(x)\sin(kx)\,dx$. The first mathematician to give a rigorous proof of anything like that claim was the German mathematician Peter Gustav Lejeune Dirichlet (1805–1859) in 1826, when he showed it was true for functions that are made up of a finite number of pieces where the function is monotonic (either increasing or decreasing) and that have only a finite number of points where their values jump. This proof invited mathematicians to investigate what happened when these conditions are broken, and to discover classes of functions with more and more disparate behavior.

Riemann, who studied under Dirichlet and learned a lot from him, was able to exhibit functions that do not have Fourier series representations, and others that agree with their Fourier series representations at only some points in their domains of definition. He found integrable functions that are discontinuous at infinitely many points in any interval. His work inspired a number of mathematicians to try to understand these new types of function, among them the German mathematicians Eduard Heine (1821–1881) and Georg Cantor (1845–1918). Heine showed that there was some hope that Fourier's claim could be proved even when a function has infinitely many points where it has jumps, provided that these jumps can be contained in intervals of arbitrarily small size. But if the function jumps at every point where $x = 1/n$, what happens at the point $x = 0$? The point $x = 0$ is a limit point of the previous ones, in the sense that any interval containing the point $x = 0$ contains a point of the form $x = 1/n$. In this case, the limit point is harmless, but there are sets whose sets of limit points can be much larger than the original set (we shall give an example below) – what about them?

Cantor was interested in the question of when a function has a unique Fourier series. He was able to show that the behavior of a point set, call it S, where a function jumps is reflected in the behavior of the set of limit points of the set S, which he called S'. Indeed, he showed that the Fourier series of a function is unique provided that one of the sets $S, S', S'', S''', \ldots$, vanishes, where S'' is the set of limit points of the set S', and so on.

Cantor gave definitions of two extreme cases involving a set and its set of limit points. If a subset S of an interval I on the real line is such that the set of its limit points S' is the whole interval I he said that S was a *dense* subset of I or an everywhere dense subset of I. If however the set of limit points S' of S is such that no interval J however small is contained in S', the set S was said to be *nowhere dense* in I. Intuitively, an everywhere dense set is unavoidable in the sense that any interval J contained in I contains limit points of S and a nowhere dense set is readily avoidable because no subinterval of I lies wholly in the limit set of S.

3.4. The Cantor Set

The famous *Cantor set*, introduced by him in 1883, is a good example of an infinite nowhere dense subset of the unit interval. It is defined iteratively. Start with all the points in the unit interval [0, 1] and throw out the points in the open middle third, (1/3, 2/3). Next, throw out the open middle thirds of the remaining two intervals, and continue in this fashion. What remains is the Cantor set. It can be best understood using what are called *ternary* 'decimals' (the analogue of decimal numbers but in base 3, not base 10). For example, the number $0.01021101122\ldots$ is such a number, and every point of the unit interval can be written as a ternary 'decimal'. Throwing out the middle third corresponds to throwing out the ternary 'decimals' that start with a 1. At the next stage those ternary 'decimals' beginning either 0.01 or 0.21 are thrown out. What is left is all those ternary 'decimals' with no 1 in their expansion. Since whole intervals are thrown out at every stage it is intuitively likely that the Cantor set is nowhere dense in the unit interval and this is indeed the case.

Interestingly, however, Cantor was confused about the implications of this set. Recall that the overriding question was the accuracy of a Fourier series representation. Mathematicians were looking for a characterization of the point sets in the real line at which a function could fail to be continuous without this affecting their Fourier series, and many examples suggested that the right characterization was that these "bad" point sets would be precisely the nowhere dense ones.

However, in a paper written in 1875, so well before Cantor's but that, unfortunately, nobody read, the Irish mathematician Henry Smith (1826–1883) had shown that this could not be so. Not only did he define a Cantor set in this paper, he gave examples to show that there are sets like the Cantor set that are nowhere dense and others that are not. This confusion could not be resolved until there was a clear distinction between the topological theory of point sets and a theory of what point sets can be ignored for the purposes of integration, and the latter had to wait for work in 1902 to 1906 with the creation of measure theory by the French mathematician Henri Lebesgue (1875–1941).

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Bibliography

Burton, D.M. (1985). *The History of Mathematics. An Introduction*, Allyn and Bacon, Inc. [An effective introduction to the history of mathematics up to about 1900.]

Fauvel, J. and Gray, J. (1987). *The history of mathematics: a reader*, Macmillan Press, Ltd. [Selection of readings of works of mathematicians through the ages outlining mathematical activities from antiquity to the twentieth century.]

Hansen, V.L. (1993). *Geometry in Nature*, A K Peters, Ltd., Wellesley, Massachusetts, U.S.A. [Geometrical view of the physical world with material on the topology of surfaces and manifolds.]

James, I.M. (ed.) (1999). *History of Topology*, Elsevier, North-Holland, The Netherlands. [A set of essays by and for mathematicians covering almost every aspect of the history of algebraic topology, but well worth consulting for further information.]

Katz, V.J. (1998). *A history of mathematics. An Introduction*, HarperCollins College Publishers, Second edition. [An excellent text with good coverage of the various periods and paying due attention to contributions from all cultures.]

Kline, M. (1972). *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York, U.S.A.[A comprehensive and excellent book on the history of the developments of mathematics with emphasis on the subjects.]

Struik, D. (1948). *A Concise History of Mathematics*, Dover Publications, Inc. (4th ed. 1987). [A classical text on the history of mathematics paying due attention to the relations of mathematics to the general cultural and sociological background of the periods.]

Biographical Sketches

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Jeremy John Gray was born on 25 April 1947 Newcastle upon Tyne, England, and is presently Professor of the History of mathematics at the Open University and an Honorary Professor in the Mathematics Department at the University of Warwick. His first degree was in Mathematics at the University of Oxford; he earned his MSc in Mathematics at University of Warwick in 1970 and his PhD in Mathematics there in 1980. He has worked at the Open University since 1974, becoming a Professor there in 2002, and has also taught at Queen Elizabeth College, London, and at Brandeis University, Waltham, Mass, USA (in 1983–1984). In autumn 1996 he was a Resident Fellow at the Dibner Institute for the History of Science and Technology, MIT, Cambridge, USA. He has lectured at the University of Warwick since 2001.

In 2009 he was awarded the Albert Leon Whiteman Memorial Prize of the American Mathematical Society for his work in the history of mathematics. He gave an Invited 45-minute Lecture at the International Congress of Mathematicians in Berlin, 1998. He has chaired Prize panels in the history of mathematics for the American Mathematical Society and the European Mathematical Society, and is one of the nine founder members of the Association for the Philosophy of Mathematical Practice.

Jeremy John Gray works on the history of mathematics, specifically the history of geometry and analysis, and mathematical modernism in the 19th and early 20th Centuries. His work on mathematical modernism links the history of mathematics with the history of science and issues in mathematical logic and the philosophy of mathematics. He is the author of six books and the co-author of two more, and he has edited or co-edited a further nine books. His most recent book is *Plato's Ghost: The Modernist Transformation of Mathematics*, Princeton University Press, Princeton 2008. His next book *The soul of the fact: a scientific biography of Henri Poincaré*, will be published by Princeton in 2012.

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Vagn Lundsgaard Hansen was Chairman Committee for Raising Public Awareness of Mathematics appointed by the European Mathematical Society, 2000–2006. He was Invited speaker International Congress of Mathematicians, Beijing 2002 and Invited regular lecturer 10th International Congress on Mathematical Education, Copenhagen 2004. President of the Jury at the 19th European Union Contest for Young Scientists, Valencia 2007. Laudatio for the Abel Prize winner Mikhael Gromov at the announcement of the Abel Prize, Oslo 2009. He is President Danish Academy of Natural Sciences since 1984, Member European Academy of Sciences 2004, President of the Danish Mathematical Society 2008–2012. Member Danish Natural Science Research Council, 1992–98, and functioned for four years in this period as vice-chairman. Member Danish Committees on Scientific Dishonesty 2003–2009.

Vagn Lundsgaard Hansen is Knight of the Order of Dannebrog and recipient of the G.A. Hagemann Gold Medal, Technical University of Denmark, for meritorious contributions to Mathematics and the Engineering Sciences.