SOCIAL CHOICE

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Summary

Arrows Impossibility implies that any social choice procedure that is rational and satisfies the Pareto condition will exhibit a dictator, an individual able to control social decisions. If instead all that we require is the procedure gives rise to an equilibrium, core outcome, then this can be guaranteed by requiring a collegium, a group of individuals who together exercise a veto. On the other hand, any voting rule without a collegium is classified by a number, v, called the Nakamura number. If the number of alternatives does not exceed v, then an equilibrium can always be guaranteed. In the case that the alternatives comprise a subset of Euclidean space, of dimension w, then an equilibrium can be guaranteed as long as $w \leq v-2$. In general, however, majority rule has Nakamura number of 3, so an equilibrium can only be guaranteed in one dimension.

1. Introduction

1.1. Rational Choice

A fundamental question that may be asked about a political, economic or social system

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is whether it is responsive to the wishes or opinions of the members of the society and, if so, whether it can aggregate the conflicting notions of these individuals in a way which is somehow rational. More particularly, is it is the case, for the kind of configuration of preferences that one might expect, that the underlying decision process gives rise to a set of outcomes which is natural and stable, and more importantly, "small" with respect to the set of all possible outcomes? If so, then it may be possible to develop a theoretical or "causal" account of the relationship between the nature of the decision process, along with the pattern of preferences, and the behavior of the social system under examination. For example, microeconomic theory is concerned with the analysis of a method of preference aggregation through the market which results, under certain conditions at least, in a particular distribution of prices for commodities and labor, and thus income. The motivation for this venture is to mimic to a degree the ability of some disciplines in natural sciences to develop causal models which tie initial conditions of the physical system to a small set of predicted outcomes. The theory of democratic pluralism is to a large extent based on the assumption that the initial conditions of the political system are causally related to the essential properties of the system. That is to say, it is assumed that the interaction of cross-cutting interest groups in a democracy leads to an "equilibrium" outcome that is natural in the sense of balancing the divergent interests of the members of the society. One aspect of course of this theoretical assumption is that it provides a method of legitimating the consequences of political decision making.

The present work directs attention to those conditions under which this assumption may be regarded as reasonable. For the purposes of analysis it is assumed that individuals may be represented in a formal fashion by preferences which are "rational" in some sense. The political system in turn is represented by a social choice mechanism, such as, for example, a voting rule. It is not assumed that society may in fact be represented in just this fashion. Rather the purpose is to determine whether such a skeleton of a political system is likely to exhibit an equilibrium. The method that is adopted is to classify all such abstract political systems. It turns out that a pure social equilibrium is very rare. This seems to suggest that if "real world" political systems are in fact in equilibrium, then what creates this equilibrium is not at all to do with what might be thought to be the initial conditions – that is to say preferences and institutional rules. Rather it must be that there are other aspects of the political system, which together with preferences and formal rules, are sufficient to generate equilibrium. It might be appropriate to use the term abstract political economy for a representation of the polity which does incorporate these further additional features.

Social choice is the theoretic discipline which is concerned with the analysis of systems of choice where the primitives are precisely preferences and rules. The development of the theory has often taken the form of focusing on certain equilibrium properties of choice mechanisms, only to find that the conditions under which they are satisfied are rare in some sense. It is useful to give a brief overview of this development of social choice, partly because there is a parallel with the structure of the current work, but also because it indicates how earlier results can be fitted into the general classification theorem for social choice mechanisms.

1.2. The Theory of Social Choice

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Social choice theory assumes that each individual i in a society $N=\{1,\ldots,n\}$ is characterized by a "rational" preference relation p_i, so that the society is represented by a profile of preference relations, $p=(p_i,\ldots,p_n)$. Let the set of possible alternatives be $W=\{x,y,\ldots\}$. If person i prefers x to y in a pair of alternatives, then write $(x,y)\varepsilon p_i$, or more commonly xp_iy. The social mechanism or *preference function*, σ, translates any profile p into a preference relation $\sigma(p)$. The point of the theory is to examine conditions on σ which are sufficient to ensure that whatever "rationality properties" are held by the individual preferences, then these same properties are held by $\sigma(p)$. Arrow's Impossibility Theorem (1951) essentially showed that if the rationality property under consideration is that preference be a weak order then σ must be dictatorial. To see what this means, let R_i be the weak preference for i induced from p_i. That is to say xR_iy if and only if (iff) it is not the case that yp_ix. Then p_i is called a *weak order* if and only if R_i is *transitive* i.e., if xR_iy and yR_iz for some x,y,z in W, then xR_iz. Arrow's theorem effectively demonstrated that if $\sigma(p)$ is a weak order whenever every individual has a weak order preference then there must be some dictatorial individual i, say, who is characterized by the ability to enforce every social choice.

It was noted some time afterwards that the result was not true if the conditions of the theorem were weakened. For example, the requirement that $\sigma(p)$ be a weak order means that "social indifference" must be transitive. If it is only required that strict social preference be transitive, then there can indeed be a non-dictatorial social preference mechanism with this weaker rationality property (Sen, 1970). To see this, suppose σ is defined by the *strong Pareto rule*: $x\sigma(p)y$ if and only if there is no individual who prefers y to x but there is some individual who prefers x to y. It is evident that σ is non-dictatorial. Moreover if each p_i is transitive then so is $\sigma(p)$. However, $\sigma(p)$ cannot be a weak order. To illustrate this, suppose that the society consists of two individuals $\{1,2\}$ who have preferences among three alternatives $\{x,y,z\}$ as follows:

1	2
x	y
z	x
y	z

This means xp_1zp_1y etc. Since $\{1,2\}$ disagree on the choice between x and y and also on the choice between y and z both x,y and y,z must be socially indifferent. But then if $\sigma(p)$ is to be a weak order, it must be the case that x and z are indifferent. However, $\{1,2\}$ agree that x is superior to z, and by the definition of the strong Pareto rule, x must be chosen over z. This of course contradicts transitivity of social indifference.

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A second criticism due to Fishburn (1970) was that the theorem was not valid in the case that the society was infinite. Indeed since democracy often involves the aggregation of preferences of many millions of voters the conclusion could be drawn that the theorem was more or less irrelevant.

However, three papers by Gibbard (1969), Hanssen (1976) and Kirman and Sondermann (1972) analyzed the proof of the theorem and showed that the result on the existence of a dictator was quite robust. Section 2.3 essentially parallels the proof by Kirman and Sondermann. The key notion here is that of a *decisive coalition*: a coalition M is decisive for a social choice function, σ, if and only if xp_iy for all i belonging to M for the profile p implies $x\sigma(p)y$. Let $\mathbb{D}_\sigma$ represent the set of decisive coalitions defined by σ. Suppose now that there is some coalition, perhaps the whole society N, which is decisive. If σ preserves transitivity (i.e., $\sigma(p)$ is transitive) then the intersection of any two decisive coalitions must itself be decisive. The intersection of all decisive coalitions must then be decisive: this smallest decisive coalition is called an *oligarchy*. The oligarchy may indeed consist of more than one individual. If it comprises the whole society then the rule is none other than the Pareto rule. However, in this case every individual has a veto. A standard objection to such a rule is that the set of chosen alternatives may be very large, so that the rule is effectively indeterminate. Suppose the further requirement is imposed that $\sigma(p)$ always be a weak order. In this case it can be shown that for any coalition M either M itself or its complement $N \backslash M$ must be decisive. Take any decisive coalition A, and consider a proper subset B say of A. If B is not decisive then $N \backslash B$ is, and so $A \cap (N \backslash B) = A \backslash B$ is decisive. In other words every decisive coalition contains a strictly smaller decisive coalition. Clearly, if the society is finite then some individual is the smallest decisive coalition, and consequently is a dictator. Even in the case when N is infinite, there will be a smallest "invisible" dictator. It turns out, therefore, that reasonable and relatively weak rationality properties on σ impose certain restrictions on the class $\mathbb{D}_\sigma$ of decisive coalitions. These restrictions on $\mathbb{D}_\sigma$ do not seem to be similar to the characteristics that political systems display. As a consequence, these first attempts by Sen and Fishburn and others to avoid the Arrow Impossibility Theorem appear to have little force.

A second avenue of escape is to weaken the requirement that $\sigma(p)$ always be transitive. For example a more appropriate mechanism might be to make a choice from W of all those *unbeaten* alternatives. Then an alternative x is chosen if and only if there is no other alternative y such that $y\sigma(p)x$. The set of unbeaten alternatives is also called the *core* for $\sigma(p)$, and is defined by

$$Core(\sigma, p) = \{x \varepsilon W : y\sigma(p)x \text{ for no } y \varepsilon W\}.$$

In the case that W is finite the existence of a core is essentially equivalent to the requirement that $\sigma(p)$ be *acyclic* (Sen, 1970). Here a preference, p, is called acyclic if and only if whenever there is a chain of preferences

$$x_0 p x_1 p x_2 p \cdots p x_r$$

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then it is not the case that $x_r p x_0$.

However, acyclicity of σ also imposes a restriction on $\mathbb{D}_\sigma$. Define the *collegium* $\kappa(\mathbb{D}_\sigma)$ for the family $\mathbb{D}_\sigma$ of decisive coalitions of σ to be the intersection (possibly empty) of all the decisive coalitions. If the collegium is empty then it is always possible to construct a “rational” profile p such that $\sigma(p)$ is cyclic (Brown, 1973). Therefore, a necessary condition for σ to be acyclic is that σ exhibit a non-empty collegium. We say σ is *collegial* in this case. Obviously, if the collegium is large then the rule is indeterminate, whereas if the collegium is small the rule is almost dictatorial.

A third possibility is that the preferences of the members of the society are restricted in some way, so that natural social choice functions such as majority rule will be "well behaved". For example, suppose that the set of alternatives is a closed subset of a single dimensional “left-right” continuum. Suppose further that each individual i has *convex* preference on W, with a most preferred point (or bliss point) x_i, say. (Convexity of the preference p just means that for any y the set $\{x : xpy\}$ is convex. A natural preference to use is Euclidean preference defined by $xp_i y$ if and only if $\| x - x_i \| < \| y - x_i \|$, for some bliss point, x_i, in W, and norm $\| - \|$ on W. This just means that a point is preferred the nearer it is to the bliss point. Clearly Euclidean preference is convex). Then a well-known result by Black (1958) asserts that the core for majority rule is the median most preferred point. On the other hand, if preferences are not convex, then as Kramer and Klevorick (1974) demonstrated, the social preference relation $\sigma(p)$ can be cyclic, and thus have an empty core. However, it was also shown that there would be a *local core* in the one dimensional case. Here a point is in the local core, $LO(\sigma, p)$, if there is some neighborhood of the point which contains no socially preferred points.

The idea of preference restrictions sufficient to guarantee the existence of a majority rule core was developed further in a series of papers by Sen (1966), Inada (1969) and Sen and Pattanaik (1969). However, it became clear, at least in the case when W had a geometric form, that these preference restrictions were essentially only applicable when W was one dimensional.

To see this suppose that there exist a set of three alternatives $X = \{x, y, z\}$ in W, and three individuals $\{1, 2, 3\}$ in N whose preferences on X are:

1	2	3
x	y	z
y	z	x
z	x	y

The existence of such a *Condorcet Cycle* is in contradiction to all the preference restrictions. If a profile p on W, containing such a Condorcet Cycle, can be found

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then there is no guarantee that $\sigma(p)$ will be acyclic or exhibit a non-empty core. Kramer (1973) effectively demonstrated that if W were two dimensional then it was always possible to construct convex preferences on W such that p contained a Condorcet Cycle. Kramer's result, while casting doubt on the likely existence of the core, did not, however, prove that it was certain to be empty. On the other hand, an earlier result by Plott (1967) did show that when the W was a subset of Euclidean space, and preference convex and smooth, then, for a point to be the majority rule core, the individual bliss points had to be symmetrically distributed about the core. These Plott symmetry conditions are sufficient for existence of a core when n is either odd or even, but are necessary when n is odd. The "fragility" of these conditions suggested that a majority rule core was unlikely in some sense in high enough dimension (McKelvey and Wendell, 1976). It turns out that these symmetry conditions are indeed fragile in the sense of being "non-generic" or atypical.

An article by Tullock (1967) at about this time argued that even though a majority rule core would be unlikely to exist in two dimensions, nonetheless it would be the case that cycles, if they occurred, would be constrained to a central domain in the Pareto set (i.e., within the set of points unbeaten under the Pareto rule).

By 1973, therefore, it was clear that there were difficulties over the likely existence of a majority rule core in a geometric setting. However, it was not evident how existence depended on the number of dimensions. Later results by McKelvey and Schofield (1987) and Saari (1997) indicate how the behavior of a general social choice rule is dependent on the dimensionality of the space of alternatives.

1.3. Restrictions on the Set of Alternatives

One possible way of indirectly restricting preferences is to assume that the set of alternatives, W, is of finite cardinality, r, say. As Brown (1973) showed, when the social preference function σ is not collegial then it is always possible to construct an acyclic profile such that $\sigma(p)$ is in fact cyclic. However, as Ferejohn and Grether (1974) proved, to be able to construct such a profile it is necessary that W have a sufficient cardinality. These results are easier to present in the case of a *voting rule* σ . Such a rule, σ, is determined completely by its decisive coalitions, $\mathbb{D}_\sigma$. That is to say:

$x\sigma(p)y$ if and only if $xp_i y$ for every $i\varepsilon M$, for some $M\varepsilon\mathbb{D}_\sigma$.

An example of a voting rule is a $q-rule$, written σ_q, and the decisive coalitions for σ_q are defined to be

$\mathbb{D}_q = \{M \subset N : |M| \geq q.$

Clearly if $q < n$ then $\mathbb{D}_q$ has an empty collegium. Ferejohn and Grether (1974) showed that if

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$q > \left(\frac{r-1}{r}\right)n$ where $|W| = r$

then no acyclic profile , p be constructed so that $\sigma(p)$ was cyclic. Conversely if $q \leq \left(\frac{r-1}{r}\right)n$ then such a profile could certainly be constructed. Another way of expressing this is that a $q-rule$ σ is acyclic for all acyclic profiles if and only if $|W| < \frac{n}{n-q}$. Note that we assume that $q < n$.

Nakamura (1979) later proved that this result could be generalized to the case of an arbitrary social preference function. The result depends on the notion of a *Nakamura number* $v(\sigma)$ for σ. Given a non-collegial family $\mathbb{D}$ of coalitions, a member M of $\mathbb{D}$ is *minimal decisive* if and only if M belongs to $\mathbb{D}$, but for no member i of M does $M \backslash \{i\}$ belong to $\mathbb{D}$. If $\mathbb{D}'$ is a subfamily of $\mathbb{D}$ consisting of minimal decisive coalitions, and moreover $\mathbb{D}'$ has an empty collegium then call $\mathbb{D}'$ a *Nakamura subfamily* of $\mathbb{D}$. Now consider the collection of all Nakamura subfamilies of $\mathbb{D}$. Since N is finite these subfamilies can be ranked by their cardinality. Define $v(\mathbb{D})$ to be the cardinality of the smallest Nakamura subfamily, and call $v(\mathbb{D})$ the *Nakamura number* of $\mathbb{D}$. Any Nakamura subfamily $\mathbb{D}'$, with cardinality $|\mathbb{D}'| = v(\mathbb{D})$, is called a *minimal non-collegial subfamily*. When σ is a social preference function with decisive family $\mathbb{D}_\sigma$ define the Nakamura number $v(\sigma)$ of σ to be equal to $v(\mathbb{D}_\sigma)$. More formally

$v(\sigma) = \min\{|\mathbb{D}'| : \mathbb{D}' \subset \mathbb{D}$ and $\kappa(\mathbb{D}') = \Phi\}.$

In the case that σ is collegial then define

$v(\sigma) = v(\mathbb{D}_\sigma) = \infty$ (infinity).

Nakamura showed that for any voting rule ,σ, if W is finite, with $|W| < v(\sigma)$ then $\sigma(p)$ must be acyclic whenever p is an acyclic profile. On the other hand, if σ is a social preference function and $|W| \geq v(\sigma)$ then it is always possible to construct an acyclic profile on W such that $\sigma(p)$ is cyclic. Thus the cardinality restriction on W which is necessary and sufficient for σ to be acyclic is that $|W| < v(\sigma)$. To relate this to Ferejohn-Grether's result for a q-*rule*, define $v(n,q)$ to be the largest integer such that $v(n,q) < \frac{q}{n-q}$. It is an easy matter to show that when σ_q is a q-rule then

$v(\sigma_{q)} = 2 + v(n,q).$

The Ferejohn-Grether restriction $|W| < \frac{n}{n-q}$ may also be written

$$|W| < 1 + \frac{q}{n-q}$$

which is the same as

$$|W| < v(\sigma_q).$$

Thus Nakamura's result is a generalization of the earlier result on q-rules.

The interest in this analysis is that Greenberg (1979) showed that a core would exist for a q-rule as long as preferences were convex and the choice space, W, was of restricted dimension. More precisely suppose that W is a compact, convex subset of Euclidean space of dimension w, and suppose each individual preference is convex and continuous. (Compactness of W just means the set is closed and bounded, while the continuity of the preference, p, that is required is that for each $x \varepsilon W$, the set $\{y \varepsilon W : xpy\}$ is open in the topology on W). If $q > \left(\frac{w}{w+1}\right)n$ then the core of $\sigma(p)$ must be non-empty, and if $q \le \left(\frac{w}{w+1}\right)n$ then a convex profile can be constructed such that the core is empty. From a result by Walker (1977) the second result also implies, for the constructed profile p that $\sigma(p)$ is cyclic. Rewriting Greenberg's inequality it can be seen that the necessary and sufficient dimensionality condition (given convexity and compactness) for the existence of a core and the non-existence of cycles for a q-rule, σ_q, is that $\dim(W) \le v(n,q)$ where $\dim(W) = w$ is the dimension of W.

Since

$$v(\sigma_{q)} = 2 + v(n,q).$$

where $v(\sigma_{q)}$ is the Nakamura number of the $q-rule$, this suggests that for an arbitrary non-collegial voting rule σ there is a *stability dimension*, namely $v^*(\sigma) = v(\sigma) - 2$, such that $\dim(W) \le v^*(\sigma)$ is a necessary and sufficient condition for the existence of a core and the non-existence of cycles.

An important procedure in this proof is the construction of a *representation* ϕ for an arbitrary social preference function. Let $\mathbb{D} = \{M_1, ..., M_v\}$ be a minimal non-collegial subfamily for σ. Note that $\mathbb{D}$ has empty collegium and cardinality $v(\sigma) = v$. Then σ can be represented by a $(v-1)$ dimensional simplex Δ in $\mathbb{R}^{v-1}$. Moreover, each of the v faces of this simplex can be identified with one of the v coalitions in $\mathbb{D}$. Each proper subfamily $\mathbb{D}_t = \{.., M_{t-1}, M_{t+1}, ..\}$ has a non-empty collegium, $\kappa(\mathbb{D}_t)$, and each of these can be identified with one of the vertices of Δ. To each $i \ \varepsilon \kappa(\mathbb{D}_t)$ we can assign a preference p_i, for $i = 1, \ldots v$ } on a set $x = \{x_{1,} x_2, \ldots, x_v\}$ giving a *permutation* profile

$$
\begin{array}{cccc}
\kappa(\mathbb{D}_1) & \kappa(\mathbb{D}_2) & \dots & \kappa(\mathbb{D}_v) \\
x_1 & x_2 & & x_v \\
x_2 & x_3 & & x_1 \\
. & . & & . \\
. & . & & . \\
. & . & & . \\
x_v & x_1 & \dots & x_{v-1}
\end{array}
$$

From this construction it follows that
$x_1\sigma(p)x_2\cdots\sigma(p)x_v\sigma(p)x_1.$

Thus whenever W has cardinality at least v, then it is possible to construct a profile p such that $\sigma(p)$ has a permutation cycle of this kind. This representation theorem is used in Chapter 4 to prove Nakamura's result and to extend Greenberg's Theorem to the case of an arbitrary rule.

The principal technique underlying Greenberg's theorem is an important result due to Fan (1961). Suppose that W is a compact convex subset of a topological vector space, and suppose P is a correspondence from W into itself which is convex and continuous. (The continuity of the preference, P, that is required is that for each $x\varepsilon W$, the set $P^{-1}(x)=\{y\varepsilon W : x\varepsilon P(y)\}$ is open in the topology on W.). Then there exists an "equilibrium" point x in W such that $P(x)$ is empty. In the case under question if each individual preference, p_i, is continuous, then so is the preference correspondence P associated with $\sigma(p)$. Moreover, if W is a subset of Euclidean space with dimension no greater than $v(\sigma)-2$, then using Caratheodory's Theorem it can be shown that P is also convex. Then by Fan's Theorem, P must have an equilibrium in W. Such an equilibrium is identical to the core, $Core(\sigma, p)$.

On the other hand, suppose that $\dim(W)=v(\sigma)-1$. Using the representation theorem, the simplex Δ representing σ can be embedded in W. Let $Y=\{y_1,\dots,y_v\}$ be the set of vertices of Δ. As above, let $\{\kappa(\mathbb{D}_t) : t=1,\dots,v\}$ be the various collegia. Each player $i\varepsilon\kappa(\mathbb{D}_t)$, is associated with the vertex y_t and is assigned a "Euclidean" preference of the form xp_iz if and only if $\| x-y_i \| < \| z-y_i \|$. In a manner similar to the situation with W finite, it is then possible to show, with the profile p so constructed, that for every point z in W there exists x in W such that $x\sigma(p)z$. Thus the core for $\sigma(p)$ is empty and $\sigma(p)$ must be cyclic. In the case that W is compact, convex, and preference is continuous and convex, then a necessary and sufficient condition for the existence of the core, and non-existence of cycles is that $\dim(W)\le v^*(\sigma)$, where $v^*(\sigma)=v(\sigma)-2$ is called the *stability dimension*. This result was independently obtained by Schofield (1984a,b) and Strnad (1985) and will be called the Schofield Strnad Theorem.

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This results on the Nakamura number can be extended by showing that even with non-convex preference, a *"critical" core* called $IO(\sigma, p)$, which contains the *local core*, $LO(\sigma, p)$, will exist as long as $\dim(W) \leq v^*(\sigma)$. It is an easy matter to show that for majority rule $v^*(\sigma) \geq 1$, and so this gives an extension of the Kramer-Klevorick (1974) Theorem.

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Norman Schofield is Director of the Center in Political Economy and William R. Taussig Professor of Political Economy at Washington University in Saint Louis. He is also Professor of Political Science. He was born in Rothesay, Scotland in 1944. After taking B.Sc degrees in physics and mathematics, in 1965 and 1966, from Liverpool University, he obtained his PhDs in Government and then Economics from Essex University in 1976 and 1985, and later a PhD and a LittD from Liverpool University in 1986. In 1991 he was awarded the Doctorat d'Etat en Science Economique from the University of Caen.

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