MATHEMATICAL MODELING IN METEOROLOGY AND WEATHER FORECASTING

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Contents

Summary

Weather forecasting is a kind of scientific and technological activity, which contributes to social and economic welfare in many sections of the community to-day. In this context, the purpose of weather forecasting is to provide information on the expected weather with forecast projection times ranging from a few hours to a few months. Almost all currently adopted forecasting techniques involve use of prediction models based on application of compressible fluid mechanics equations to the atmosphere. The models provide information necessary not only for the weather prediction but also for many other applications (aviation, ship routing etc.). As a rule, the models are running within computerized real-time forecasting systems which involve automated collection, checking, and numerical analysis of observations necessary for weather forecasting. According to the space-time scales of predicted weather systems, to completeness and quality of available input data, and to the capability of the data communication and processing equipment, the models differ in scales of simulated weather-creating processes and in the accounting for various physical factors influencing the evolution of meteorological fields. All these principles are applied to modeling, development (evolution) and formation (genesis) of tropical cyclones (typhoons, hurricanes). Depending on the problem to be solved tropical cyclone modeling may differ in what concerns the set of equations, parameterization of physical processes involved, and the form of the initial vortex representation.

Within this general approach, the particular techniques for local weather forecasting may involve means for statistical treatment of the model output in terms of weather parameters. The techniques used for very short-range forecasting of local-scale hazardous weather phenomena (tornadoes, heavy showers, local thunderstorms etc.) are based on application of the forecaster-computer interactive systems with the use of real-time satellite and radar observations, and of the output provided by mesoscale dynamic models.

Advances in the atmospheric sciences and in atmospheric modeling as well as in the development of weather observation systems and automated data processing and computation techniques, have resulted in immutable rise of the weather forecasting skill, which has been going since the implementation of first dynamical forecasting models in the 1950s.

1. Introduction

This Chapter provides an outline of atmospheric mechanisms responsible for the formation and evolution of weather patterns in both large areas and particular sites, i.e., the meteorological fields of air temperature and humidity, wind, cloudiness, and precipitation. To give an understanding of how the weather patterns are formed, it is necessary to explain physical phenomena behind the various atmospheric processes and motions.

Meteorological processes do develop in the atmosphere, a gas envelope of earth, which consists of nitrogen, oxygen, water vapor, carbon acid etc. The atmosphere, a gas envelope of the earth, is in perpetual motion and interaction with the underlying surface.

In the atmosphere, processes go on at all times transforming the solar radiation energy into various types of thermal energy and into the energy of motions, whose very broad spectrum ranges from planetary-scale motions to small-amplitude turbulent pulsations of atmospheric flows. All such processes take place according to three fundamental physical laws governing all geophysical processes: the momentum, mass, and energy conservation laws. A distinctive feature of the weather-producing processes lies in their occurrence on the rotating earth and within thermally-stratified atmosphere - a compressible baroclinic fluid. The atmosphere rotates together with our planet; therefore in the non-inertial (rotating) frame fixed to the earth, all atmospheric motions are subject to the action of a deflection force referred to as the Coriolis force. This force, which affects all moving air particles in the atmosphere and water particles in the oceans, seas, and rivers, and modifies their motion, depends on latitude and decreases with distance from the equator. This special feature of processes in the air and water envelopes of earth has led to setting off from the hydrodynamics and earth sciences of a separate branch of science - the geophysical fluid dynamics. The other special feature of atmospheric processes consists in their occurrence within a stratified fluid. Due to this, the atmospheric flows are baroclinic, i.e. unlike the barotropic ones the air density in the atmospheric flows depends not only on pressure but also on temperature.

Special phenomena studied in the geophysical fluid dynamics manifest themselves primarily in the atmospheric processes whose scales are comparable to the planetary scale like the earth’s radius (about 6000 km). Atmospheric processes just of this scale are responsible for the formation and evolution of planetary waves and related atmospheric mechanisms creating weather systems and weather patterns over large areas, like continents and oceans. Exploring and mathematical modeling of such large-scale motions provides a means for development of numerical methods applicable to the weather forecasting of various lead times.

A distinctive feature of motions in the earth’s atmosphere and hydrosphere lies in their predominantly turbulent nature, which deserves consideration in geophysical fluid dynamics. This is provided by employment of a model of geophysical turbulence which treats the turbulence in rotated stratified fluids. In this respect, the geophysical turbulence model differs significantly from the models commonly used for treatment of the turbulence of other types, e.g. those inherent in motion of a fluid in industrial devices. Principles and methods of the geophysical turbulence model are widely used in the mathematical models for weather forecasting.

Atmospheric processes and motions of smaller scales do develop against the background of large-scale processes and contribute to shaping of the regional and local weather patterns. These processes, which are referred to as mesometeorological processes, depend on the local features of the underlying surface like local orography, landscape, albedo etc. Finally, the processes of still smaller scales occur and develop over small areas or within various layers of the atmosphere. These are the convection, turbulent mixing in the atmospheric boundary layer, phase conversions of water in the air and on the land and sea underlying surface etc. In some cases the contribution of these processes to the local weather may be rather significant; however the impacts of these processes differ in both space and duration. Therefore in some cases they should be taken into account explicitly, while in the other cases they may be neglected or calculated approximately as corrections; the latter procedure of accounting for small-scale processes is referred to as parameterization.

Exploring and mathematical modeling of weather-producing mechanisms and weather patterns by the methods of the geophysical fluid dynamics show that the main causes of the weather evolution are associated with the occurrence and evolution of the large-scale atmospheric vortices and also on their life time. The large-scale atmospheric disturbances are supplemented with fast moving vortices of smaller scales like tropical cyclones (hurricanes, typhoons). These vortices (their modeling and forecasting is also within the scope of this chapter) may develop huge energy and winds; this is the main cause of their exceptionally great threat to human life and damage to the property.

In accordance with all of the preceding, this chapter gives a description of atmospheric models which provide a means for the weather forecasting of various projection times. These are:

- large-scale forecast models,
- mesoscale models,
- atmospheric boundary layer models, and
- tropical cyclone models.

Some of the above small-scale atmospheric processes (e.g., convection) are not described here. However the reader may gain knowledge related to them from other appropriate chapters of this Encyclopedia. The same is true for the model of geophysical turbulence.

2. Equation System used in Hydrodynamic Atmospheric Models

2.1. General Formulation

Mathematical formulation of atmospheric models used for weather forecasting is based on the equations of mechanics of a compressible fluid, which stem from three fundamental laws: the laws of the momentum and mass conservation and the first law of thermodynamics. These three laws give rise to equations of motion, equation of continuity, and thermodynamic energy equation (or heat transport-transform equation), respectively. Being represented in the coordinate system fixed to rotating earth, the equation of motion in three dimensions may be written in the form

$$\frac{d\vec{V}}{dt} = -\frac{1}{\rho}\operatorname{grad} p - 2\vec{\omega}\times\vec{V} - \operatorname{grad} U + \vec{D}$$

where $\vec{V}$ is the velocity vector, d/dt is symbol of the total derivative with respect to time, ρ is density, p is pressure, $\vec{\omega}$ is the vector of angular velocity of earth's rotation, U is potential of the gravity force, and $\vec{D}$ is the vector representing dissipation forces. The equation of continuity has the form

$$\frac{d\rho}{dt} + \rho \operatorname{div} \vec{V} = 0$$

and the thermodynamic energy equation is formulated as

$$\rho \frac{ds}{dt} = \frac{1}{T}\varepsilon$$

where s is entropy of a particle, T is temperature, and ε is the rate at which the heat is added to unit air mass due to external (nonadiabatic) sources of energy; these are: solar and thermal radiation, turbulent heat exchange, and phase transformation of atmospheric moisture. Since the atmosphere is regarded as ideal gas, entropy is represented as $s = C_V \ln\left(p/\rho^{\kappa}\right) + \text{const}$ where κ is the ratio of coefficients for the specific heat capacity of air at constant pressure and constant volume. T is linked with p and ρ with the Boyle-Charles equation of state for ideal gas: $p = R\rho T$ where R is the gas constant for dry (unsaturated) air. The above equations are supplemented with equation for transfer of specific humidity, q,

$$\frac{dq}{dt} = Q$$

where Q describes both the turbulent diffusion of humidity and the release of water in the form of precipitation.

In the local Cartesian coordinates x, y, and z the above equations are:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f\upsilon - f_1 w$$

$$\frac{d\upsilon}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\frac{d\ln\left(p/\rho^{\kappa}\right)}{dt} = 0$$

where u, υ, w are the velocity components in x, y, z directions, f is the Coriolis parameter: $f = 2\omega\sin\varphi$, $f_1 = 2\omega\cos\varphi$, g is the upward component of gravitational acceleration, φ is latitude, and

$$\frac{\mathrm{d}\psi}{\mathrm{dt}}=\frac{\partial\psi}{\partial \mathrm{t}}+\mathrm{u}\frac{\partial\psi}{\partial \mathrm{x}}+\upsilon\frac{\partial\psi}{\partial \mathrm{y}}+\mathrm{w}\frac{\partial\psi}{\partial \mathrm{z}} \qquad \left[\ \psi=\mathrm{u},\ \upsilon,\ \mathrm{w},\ \rho,\ \ln\left(\mathrm{p}/\rho^{\kappa}\right)\ \right] \ .$$

2.2. Hydrostatic Relation

An important point in the application of the above equations in weather forecasting lies in the assessment of characteristic magnitudes of their particular terms.

It is a matter of experience that atmospheric motions which determine the weather-producing mechanisms are concentrated in the lower atmospheric layer with vertical depth of an order of 10-20 km. But the horizontal extent of atmospheric processes is very variable and may range from several hundred meters (local breezes, mountain-valley winds, mountain clouds, etc.) to a few thousand kilometers (cyclones, depressions, monsoon circulation systems, etc.).

In particular, the large-scale weather patterns are associated with atmospheric motions having the scale of the order of 1000 km, i.e. the horizontal scale of the phenomena concerned are at least ten times as large as their vertical scale. As it is evident from the equation of continuity, in this case the characteristic vertical velocity, W, is at the same ratio in relation to the characteristic horizontal velocity, U, as the characteristic vertical scale of the phenomena, H, is in relation to the characteristic horizontal scale, i.e. $\mathrm{W}/\mathrm{U}\approx \mathrm{H}/\mathrm{L}$.

The characteristic time, t_1, may be taken as $\mathrm{t}_1=\mathrm{L}/\mathrm{U}$ (or $\mathrm{t}_1=\mathrm{H}/\mathrm{W}$). This means that the rate of propagation of a phenomenon is assumed to be the same as the speed of the air motion, which in turn implies that the propagation of atmospheric disturbances does not exhibit properties of a process whose propagation rate is considerably greater than the transference rate of air particles, as it is the case in the propagation of the sound waves for example. If U = 10 m s^{-1}, H/L = 0.1, and H = 10 km, each of the terms in the left-hand side of the equation of motion in vertical direction is of the order of $\mathrm{U}^2\mathrm{H}/\mathrm{L}^2\approx 10^{-4}$ m s^{-2} which is 10^{-4} times smaller than g in its right-hand side. Hence, this equation simply states that gravitational acceleration, g, is almost exactly balanced with acceleration due to vertical gradient of atmospheric pressure (or atmospheric buoyancy), $\rho^{-1}\partial\mathrm{p}/\partial\mathrm{z}$, and may be replaced with the *hydrostatic* (or *quasistatic*) relation

$$-\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{z}}+\mathrm{g}=0$$

for a wide range of weather-creating atmospheric processes. However this relation is not fulfilled even approximately if the vertical and horizontal scales of some weather phenomena are of the same order of magnitude. Hence the modeling of certain local weather phenomena calls for employment of complete (non-simplified) equation of motion in the vertical direction. Atmospheric phenomena of this type are predicted with *mesoscale* atmospheric models discussed in Sub-section 4.2.

2.3. Equation System in Pressure Coordinates

Quasistatic relation provides a means to bring the forecast equations used for prediction of large-scale atmospheric patterns into a simpler form which is based on *pressure coordinates* developed by Eliassen (1948). In the system of pressure coordinates, the usual x, y coordinates denote a point's position projected into a horizontal plane, but the pressure p denotes its location along the vertical axis. The "horizontal"derivatives of a variable are its differences from one point to another *in the same isobaric surface* with respect to corresponding differences *in horizontal plane*. The "vertical" derivative of a variable is its derivative with respect to pressure, but is directed along the *vertical* axis. In pressure coordinates pressure p becomes one of the independent variables, the height z of a particular isobaric surface becomes a dependent variable, and the role of the vertical air speed w is taken over by ω, the total derivative of pressure with respect to time: $\omega = dp/dt$.

Equations of motion in horizontal directions, hydrostatic equation, equation of continuity, and thermodynamic energy equation in p-coordinates are:

$$\frac{du}{dt} = -g\frac{\partial z}{\partial x} + f\upsilon ,$$

$$\frac{d\upsilon}{dt} = -g\frac{\partial z}{\partial y} - fu ,$$

$$T = -\frac{g}{R}\frac{p\,\partial z}{\partial p} ,$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} + \frac{\partial\omega}{\partial p} = 0 ,$$

and

$$\frac{dT}{dt} - \frac{\kappa - 1}{\kappa}\frac{T}{p}\omega = 0 ,$$

where $d\psi/dt = \partial\psi/\partial t + u\partial\psi/\partial x + \upsilon\partial\psi/\partial y + \omega\partial\psi/\partial p$ ($\psi = u, \upsilon, T$). The relationship between w and ω is

$$w = \frac{\partial z}{\partial t} + u\frac{\partial z}{\partial x} + \upsilon\frac{\partial z}{\partial y} - \frac{\omega}{g p} \approx -\frac{\omega}{g p} .$$

The equation set in p-coordinates is simpler than the initial one. Thus, the equations of motion and equation of continuity resemble the analogous equations for an incompressible fluid. Besides, in constructing upper-air charts in meteorological practice just p is used as independent variable and the height z of isobaric surfaces as functions being analyzed. Together with further modifications intended for convenient treatment of the relief impacts, p-coordinates are widely used the geostrophic and non-geostrophic models nowadays for prediction of large-scale weather patterns. The geostrophic and non-geostrophic models are discussed in Sub-sections 3.3 and 4.1, respectively.

3. Hydrodynamical Modeling of Large-scale Weather-producing Mechanisms

This section provides a description of approaches and methods applicable to hydrodynamic forecasting of atmospheric processes responsible for formation of large-scale weather patterns. Sub-section 3.1 deals with analysis of possible atmospheric wave-like motions in the context of their simulation with hydrodymamic equations. The large-scale atmospheric waves: determining evolution of extratropical weather systems on the time scale of several days, the atmospheric Rossby waves, are described in Sub-section 3.2. Finally, Sub-section 3.3 deals with quasi-geostrophic atmospheric models applicable to approximate short- and medium-range forecasting of synoptic-scale weather systems.

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Biographical Sketches

Berkovich Leopold Vladimirovich, was born in the Ukraine (city of Kirovograd) on 27 January 1937. In 1956 he entered the Odessa Hydrometeorological Institute (the Meteorological Faculty) and graduated in 1960 as engineer-meteorologist. In 1961 he entered the postgraduate courses of the Central Institute of Forecasts (now the Hydrometeorological Research Center of the Russian Federation) specializing in numerical methods of weather forecasting. In 1966 he received the Ph. D. degree for his work on " Numerical forecasting of meteorological fields". Since 1964 he is working in the above Center , since 1994 as Chief of Division of Hydrometeorological Short – range Weather Forecasts. The scope of his basic scientific interests covers the numerical prediction of meteorological fields and weather forecasts for individual locations (main cities etc.). He has developed an operational hemispheric numerical model for the prediction of meteorological fields and weather elements . He authored about 70 scientific papers as well as several chapters (in co- authorship) in some training publications and monographs. He lectured on the above subjects in the universities. Under his leadership 3 PhD theses have been prepared and defended.

Sitnikov Igor Georgievich, was born in Moscow on 11 March 1935. In 1953 he entered and in 1959 graduated from the Physical School of the Moscow State University. Since 1959 he has been working in Central Institute of Forecasts (Moscow), which later was renamed as the Hydrometeorological Research Center of the Russian Federation. His present position is the Chief of Branch named "Branch of Atmospheric Dynamics in Tropical Zone". In 1971 (for one year) and in 1977-1981 he worked as a consultant and a Senior Officer in the Secretariat of the World Meteorological Organization (Geneva, Switzerland) , grade P5, dealing with the administrative and scientific coordination of the GARP Atlantic Tropical Experiment (GATE), whose field phase was carried out in 1974, and the First GARP Global Experiment (FGGE) fulfilled in 1978-1979. Besides that, he participated in the many national and international scientific conferences, symposia and workshops primarily associated with studies in tropical meteorology and , in particular, tropical cyclones. He developed several numerical models including a barotropic model for the prediction of geopotential in the middle latitudes and several models for the prediction of tropical cyclone tracks and development. He earned the degrees of PhD and full Doctor of Sciences and an academic title of Professor.

Shnaidman Volf Abramovich, was born in the Ukraine (city of Chernovci) on 19 January 1934. In 1952

He entered the Odessa Hydrometeorological Institute (the Meteorological Faculty) and graduated in 1957 as engineer-meteorologist. From 1961 to 1999 he was with the Odessa Hydrometeorological Institute . Since 2000 he is a Visiting Professor of the Department of Environmental Science of New Jersey State Rutgers University (USA). He earned the degrees of PhD , Doctor of Sciences and an academic title of Professor. He authored about 80 scientific papers and 3 monographs. Under his scientific leadership 8 PhD theses have been prepared and defended.