COMPUTATIONAL TECHNIQUES

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Contents

Summary

The chapter describes different computational techniques which are useful for design and analysis of pressure vessels and piping systems as well as various other engineering applications. The techniques presented here are the finite element method, boundary element method, element free method, finite volume method, and lattice Boltzmann method. Because most of those techniques can be developed from the weighted residual method, the method is also presented. These techniques can be used for structural integrity of pressure vessels and pipes, flow analysis, heat transfer, and multi-media interaction. Traditionally, some have been more frequently used to solid and structural system, and others for fluid and thermal system. In addition, some techniques are at the more mature stages than others which have been developed more recently. It would be beneficial to understand those different techniques and apply them as needed for design and analysis of reliable and complex pressure vessels and piping systems.

1. Introduction

Design and analysis of pressure vessels and piping components and a system requires accurate and reliable calculations of necessary solutions such as stresses, strains, natural frequencies, mode shapes, fluid pressures and velocities, temperatures, etc. in order to achieve the desired performance of the system as well as to avoid any unexpected failure which may be catastrophic for some systems such as nuclear power systems. As a result, the computational techniques have been used extensively for such designs and analyses. In particular, the computational techniques provide engineers with physics-based modeling and simulation capabilities so that expensive and sometime risky experimental tests can be avoided or at least minimized.

For the pressure vessels and piping communities as well as for most of other industrial communities, two computational techniques have been used almost exclusively. They are the finite element method and the finite volume method. The finite element method has been used extensively for stress and dynamic analyses while the finite volume method has been used more frequently for thermal-fluid analyses. Therefore, those techniques are discussed in this chapter. In addition, this chapter also presents other techniques which have not been as popular as the two mentioned techniques so far but have great potential for more frequent uses in the near future. Those techniques include boundary element method, element free method, and lattice Boltzmann method. The first two are useful for analyses of the solids and structures while the last one is good for the fluid medium.

Understanding and comparing those techniques will be beneficial for improvement of the design and analysis procedures of pressure vessels and piping components and systems. The subsequent section discusses the weighted residual method which has played an important role in developing various computational techniques. Then, different computational techniques are presented one after another. Finally, conclusions are provided at the end of this chapter.

2. Weighted Residual Method

Most of real life engineering problems are too complex to obtain their exact solutions. As a result, it is necessary to determine acceptable approximate solutions to those problems. With the aid of computing technology in terms of hardware and software, various computational techniques have been developed to provide reliable approximate solutions to most of complicated engineering problems including those in pressure vessels and piping systems. Those problems include, but not limited to, linear and nonlinear stress analyses, buckling analyses, vibration analyses, fluid flows and heat transfer analysis, multi-media analysis such as fluid-structure interaction, etc.

Many of computational methods can be derived from approximate analytical solution techniques. One of such techniques is the weighted residual method. This technique is based on an assumed function of approximate solution set which contains some unknown coefficients. This assumed function is called a trial solution. For a better explanation, we will consider the following problem which is the 2^{nd} order ordinary differential equation:

$$\frac{d^2u}{dx^2} - 1 = 0, \ 0 < x < 1, u(0) = 0 \text{ and } u(1) = 0 \qquad (1)$$

Even though the exact solution is available for the given problem, we will demonstrate how to obtain an approximate solution using the weighted residual formation. As stated above, a set of approximate solution is assumed. Because the polynomial function is the simplest form of function and it can represent any complex function using the polynomial series expansion, a polynomial function is usually selected as an approximate solution set.

For the given problem, the lowest order of polynomial for an assumed function is a quadratic polynomial function. If a linear polynomial were selected as a trial solution to meet the given boundary conditions, the solution would be zero. As a result, a quadratic polynomial is the lowest order of a polynomial trial solution with a non-zero solution. That is, the trial solution is chosen as

$$\tilde{u} = ax^2 + bx + c \qquad (2)$$

where $a, b,$ and c are coefficients of the polynomial, and $\tilde{u}$ represents the trial solution. Applying the boundary conditions to the trial solution yields

$$u = ax(x-1) \qquad (3)$$

Now, the trial solution has one coefficient to be determined. The eventual approximate solution depends on the coefficient which should be should be determined in order to make the approximate solution as close as possible to the exact solution. In order to determine an optimal value for the coefficient, we need to have a measure for the error. One of such measures is the residual which is computed by substituting the trial solution into the differential equation. Then, the residual is expressed as

$$R = 2a - 1 \qquad (4)$$

If the residual vanishes over the whole problem domain, the approximate solution becomes the exact solution because it satisfies both the differential equation and the boundary conditions exactly. However, in most of cases the residual is not zero all over the problem domain even though it may be zero at some selected locations of the problem domain. This means the approximate solution is not the exact solution. We also want to make the approximate solution as close as possible over the entire problem domain rather than at certain local zones. Therefore, we want to minimize the sum of the errors. In that aspect, we need to compute the sum of residual over the entire domain. This means integration the error over the problem domain.

In order to avoid canceling a positive error by a negative error, we add up the square of the residual over the problem domain as given below:

$$I(a) = \int_0^1 \{R(a,x)\}^2 dx \qquad (5)$$

Then, we minimize the sum of the squared residual with respect to the unknown coefficient a by taking a derivative as below:

$$\frac{dI(a)}{da} = 2\int_0^1 \frac{dR}{da} R dx = 0 \qquad (6)$$

Solving the above equation yields the value for the unknown coefficient $a = 0.5$. This means an optimal approximate solution is

$$\tilde{u} = 0.5x(x-1) \quad (7)$$

We may generalize Eq. (6) into the following expression:

$$I = \int_0^1 w(x)R(x)dx = 0 \quad (8)$$

In this equation, the residual was multiplied by a weight function (or called a test function). Then, the weighted residual was summed over the problem domain and set equal to zero. Comparing Eqs. (6) and (7) results in $w = dR/da$ because the constant value 2 does not affect the solution. When the test function is selected in such a way, it is called the least square method that is one of the weighted residual formulations. In general, the test function may be selected differently. Depending on its choice, the weighted residual method can be classified into various techniques. One of the common techniques is the Galerkin method for which the test function is chosen as

$$w = \frac{d\tilde{u}}{da} \quad (9)$$

Applying the Galerkin method to the previous example gives $w = x(x-1)$. Then, the coefficient a is determined from Eq. (8), and it turns out to be 0.5. In this case, both the least squares method and the Galerkin method resulted in the same value for a because their solutions are the exact solution. However, the two methods, in general, yield different approximate answers.

If we want to improve the accuracy of the approximate solution, we may use a higher order polynomial function for the trial function. For example, if a cubic polynomial function is selected for the trial function instead of a quadratic polynomial function, there are two coefficients to be optimized after applying the two boundary conditions. Because the quadratic function is a subset of the cubic function, the cubic function has a greater flexibility to represent the solution with less minimal error. As the order of the polynomial function increases, the accuracy of the solution also improves. If the exact solution can be expressed in a polynomial of certain order, and the trial function is selected with the same or higher order of polynomial, the approximate solution will turn out to be the exact solution because the exact solution has the least of all the minimal errors, i.e. the zero error.

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Biographical Sketch

Young W. Kwon was born in Korea. He received a BS degree in mechanical engineering from Seoul National University in Seoul, Korea in 1981, and a PhD degree in the same field from Rice University in Houston, Texas, USA in 1985. He was a faculty member at the University of Missouri-Rolla and Southern Illinois University. Presently, he is a professor of mechanical and astronautical engineering department of US Naval Postgraduate School, Monterey, California, USA. He was also the department chair at Southern Illinois University as well as at Naval Postgraduate School. He has a very extensive knowledge and experience in computational techniques including multi-scale modeling and simulation techniques. He published more than 200 technical works. He also authored textbooks, *Finite Element Method Using Matlab* (1st ed. 1996; 2nd ed, 2000) published by CRC Press (Florida, USA), and edited a book called *Multiscale Modeling and Simulation of Composite Materials and Structures* (2007) published by Springer (New York, USA). He received various honors and awards including Cedric Ferguson Medal from Society of Petroleum Engineers, Best Research Award from American Orthopedic Society of Sports Medicines, Menneken Research Award from Naval Postgraduate School, and Outstanding Service Award from American Society of Mechanical Engineers (ASME). He is a fellow of the ASME.

Dr. Kwon has been active in professional societies. In particular, he is an Executive Committee member (Communication Chair) of Pressure Vessels and Piping Division (PVP) of ASME. He is also editor of the PVP Division Newsletter and conference proceedings. He will be the division chair during 2010-2011, and the annual PVP Conference Chair during 2009-2010. He also served as the Fluid-Structure Interaction Committee and the associate editor of *Journal of Pressure Vessel Technology*.