

## SOIL MECHANICS

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## Summary

This chapter describes briefly the content of the discipline called soil mechanics. At the beginning, definition of several property indices commonly in use in practice is introduced together with inter-relation amongst them. Secondly, simple tests necessary for conducting engineering classification of soils are introduced. These are sieve analysis and consistency tests. The next subject is the concept of artificially compacting soils for various purposes including subgrade construction for highways, railways or earthworks for rock-fill dam construction. Seepage of water through soils is an important subject when evaluating an amount of leaking water or in assuming stability or instability of soil deposits undergoing seepage flow: Consolidation of clays is the theme associated with time-progressing compression of soil deposits leading to settlements of wide terrains.

In order to assess stability or instability of various kinds of slopes and also for the purpose of evaluating the magnitude of earth pressures against wall structures and for estimating the bearing capacity of foundations, it is of essential to be able to assess the shear strength of soils. The basic concept and its adaptation to different water-drainage conditions are introduced to evaluate shear strength of soils relevant to each situation. Some test data are also presented. Lastly, as the main areas in which the science of soil mechanics is widely applied, the theory of earth pressure, slope stability and bearing capacity are introduced to help understand how the basic knowledge of soil mechanics is applied to major issues of practical importance.

### 1. Introduction

Soil mechanics deals with sediments and any other accumulations of solid particles produced by mechanical and chemical disintegration of rocks. Organic materials such as peat existing near the ground surface are also the objects of studies. The laws of continuum mechanics and hydraulics are the basic fields from which the soil mechanics has evolved. The major aspects considered in soil mechanics would be the deformation and strength characteristics and seepage flow through soils. There are many areas of application of this knowledge for engineering purposes including rational design of foundation, construction of earth structures such as embankments and rockfill dams, construction of underground structures such as tunnels. Mitigation of geotechnical hazards such as landslides, earthquake-induced liquefaction, flooding and volcanic activities also requires ample knowledge of the behavior of various kinds of soil deposits. The following sections provide a brief introduction to the knowledge of basic importance in understanding soil behavior constituting the surface of the earth.

### 2. Definition of Property Indices

There are many indices and parameters of soils being used to represent physical properties of soils. Only those which are of most importance and of frequent use will be introduced here.

#### 2.1. Definition of Basic Property Indices

The soils in natural and man-made deposits consist of three phases: solid, fluid and air. To express proportions in volume of fluid or air relative to the solid part, the following property indices are used.

**2.1.1. Void Ratio,  $e$**

The void ratio is defined as the proportion of void (fluid plus air) relative to the volume of solid phase.

$$e = \frac{V_w}{V_s} \tag{1}$$

where  $V_w$  and  $V_s$  denote, respectively, the volume of void and solid phase consisting mainly of mineral substance, as illustrated in Figure 1. For sands, the void ratio takes values between 0.5 and 1.2 and for silts the void ratio ranges between 1.0 and 1.5. The void ratio takes values for a majority of clays ranging widely between 1.5 and 3.0. It is to be noticed that the void ratio tends to increase with decreasing grain size.

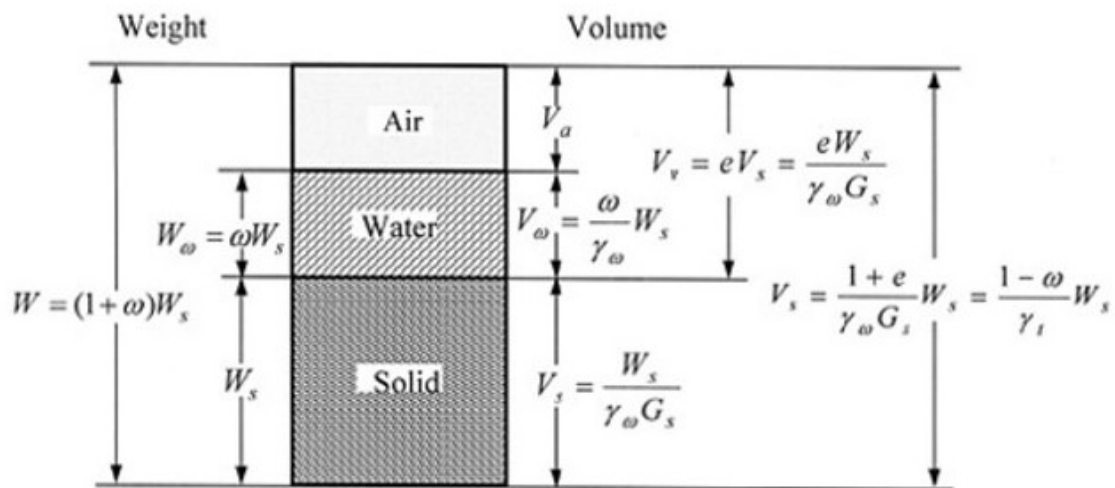


Figure 1: Weight and volume of each phase as a function of the weight  $W_s$  of the solid phase

**2.1.2. Saturation Ratio,  $S_r$  (%)**

The proportion of the volume of water,  $V_w$  to that of void,  $V_v$ , defines the saturation ratio,  $S_r$  as

$$S_r = \frac{V_w}{V_v} \tag{2}$$

According to the definition, dry soils without water in the void have a saturation ratio of  $S_r = 0\%$ , whereas soils fully saturated with water have a saturation ratio of 100%.

### 2.1.3. Water Content, $\omega$ (%)

With reference to Figure 1, the proportion in weight of fluid,  $W_w$ , relative to that of the solid phase,  $W_s$ , is expressed in terms of the water content which is defined as

$$\omega = \frac{W_w}{W_s} \quad (3)$$

### 2.1.4. Bulk or Wet Unit Weight $\gamma$ or $\gamma_t$

The total weight,  $W$ , (solid plus fluid) divided by the total volume,  $V$ , of a soil is termed bulk unit weight or wet unit weight and defined as

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} \quad (4)$$

Generally speaking, the wet unit weight of soils changes between  $15\text{kN/m}^3$  and  $20\text{kN/m}^3$ .

### 2.1.5. Dry Unit Weight $\gamma_d$

The weight of solid phase  $W_s$  divided by the total volume,  $V$ , of a soil is termed dry unit weight and defined as,

$$\gamma_d = \frac{W_s}{V} \quad (5)$$

According to the definition, the dry unit weight of soils is always smaller than the wet unit weight. The value of  $\gamma_d$  is used as a measure of looseness or denseness of soils.

### 2.1.6. Unit Weight and Specific Weight of the Solid Phase $\gamma_s$ and $G_s$

The unit weight of the solid phase is defined as

$$\gamma_s = \frac{W_s}{V_s} \quad (6)$$

Similarly, the unit weight of the water is defined as

$$\gamma_w = \frac{W_w}{V_w} \quad (7)$$

The ratio of the unit weight of the solid phase to that of the water phase is known as the specific weight of the solid

$$G_s = \frac{\gamma_s}{\gamma_w} \quad (8)$$

Since a majority of soil particles consists of crushed rocks, the value of  $G_s$  takes a value between 2.5 and 2.7.

### 2.1.7. Buoyant Unit Weight or Submerged Unit Weight, $\gamma'$

When a soil exists in the deposit below the ground water table, or in under-water environment such as the seabed or lakebed, there is no air contained in the void. In such a case, the soil is referred to as saturated with water. It often becomes necessary to estimate the inter-granular stress at a given depth which is generated by the weight of overlying soils submerged under water. For this purpose, the submerged unit weight plays a key role.

The solid particle submerged in water is known to reduce its weight by an amount,  $\gamma_w V_s$ , which is equal to the weight of water expelled by the presence of the solid. Thus, the submerged weight is  $W_s - \gamma_w V_s$  and the submerged unit weight is defined as

$$\gamma' = \frac{W_s - \gamma_w V_s}{V} = \frac{W - W_w - \gamma_w V_s}{V} = \frac{W - \gamma_w (V_w + V_s)}{V} = \frac{W}{V} - \gamma_w = \gamma_{\text{sat}} - \gamma' \quad (9)$$

where the bulk weight in saturated soil is denoted by  $\gamma_{\text{sat}}$ .

## 2.2. Relations among Property Indices

The unit weights defined above can be expressed in terms of the void ratio,  $e$ , specific gravity,  $G_s$ , and saturation ratio,  $S_r$ , as follows

$$\gamma_t = \frac{G_s + S_r e}{1 + e} \gamma_w$$

$$\gamma_d = \frac{G_s}{1 + e} \gamma_w$$

$$\gamma_{\text{sat}} = \frac{G_s + e}{1 + e} \gamma_w$$

$$\gamma' = \frac{G_s - 1}{1 + e} \gamma_w \quad (10)$$

The property indices which can be easily determined in the laboratory or in the field with a high level of accuracy are water content,  $\omega$ , specific gravity,  $G_s$ , and bulk unit

weight,  $\gamma_t$ . Thus, in routine practices the other parameters are usually determined in sequence as follows using the above three quantities.

$$\gamma_d = \frac{\gamma_t}{1 + \omega}$$

$$e = \frac{G_s}{\gamma_d} \gamma_w - 1$$

$$S_r = \frac{\omega G_s}{e} \tag{11}$$

### 2.3. Sieve Analysis and Grain Composition

A soil is composed of a number of particles with a variety of size ranging from clay to gravel. It becomes necessary to provide practical procedures to measure distribution of particle size for a given soil and to express it in terms of index properties. For this purpose, the sieve analysis have been used to identify proportion in weight of soil particle included in each of given size ranges. It has been customary to express the grain size in terms of the mesh size of the sieve through which the soil particles can or cannot pass. The mesh size and the naming of each sieve are shown in Figure 2. The smallest mesh size used in routine practice in Japan and U.S. is 0.074 mm = 74  $\mu$  (micron) and the sieve with this mesh size is called No. 200 mesh. In some of European countries, the smaller-sized mesh with 60  $\mu$  is used. The sieves with larger-sized meshes, 0.105mm, 0.250mm, etc. denoted, respectively as No. 100, No. 60, etc. are used for the routine sieve analysis. The sieve with a mesh size of 4.76mm is called No. 4 sieve and the sieves with larger mesh size are denoted in terms of inch as accordingly indicated in Figure 2.

Particle size	10 $\mu$		100 $\mu$			1mm		10mm		
Mesh size	74 $\mu$		105 $\mu$	250 $\mu$	420 $\mu$	840 $\mu$	4.76mm	9.52mm	19.1mm	25.4mm
No. of sieves	5 $\mu$	#200	#100	#60	#40	#20	#4	3/8 inch	3/4 inch	1 inch
Name	Clay	Silt	Fine sand	Medium sand	Coarse sand		Gravel			
	Clay	Silt	Sand				Gravel			
1 $\mu$ (Micron) = 10 <sup>-3</sup> mm    #:No. of sieves : (1 inch=2.54cm)										

Figure 2: Classification in terms of the particle size

According to the Unified Classification System adopted in the U.S., the particles posing No. 4 sieve but remaining in No. 200 sieve are called “sand”, and those remaining in No. 4 mesh is termed “gravel”. The particles passing No. 200 sieve is generally called “fines”. The fines are further divided into clay and silt portion as indicated in Figure 2. For identifying the grading or grain composition of the fine-grained soils passing the

No. 200 mesh, another type of test using the Picnometer is conducted, but in the majority of routine practice, this phase of the test is omitted.

Mechanical properties of coarse-grained soils composed of sand and gravel are generally dominated by the grain composition and thus it has been customary to perform the sieve analysis for the sand and gravel and to obtain the index properties. For fine-grained soils, what is called “Consistency test” has been in use for identifying engineering properties of the soils.

### 2.3.1. Grain Composition

In the sieve analysis in the laboratory, five to six sieves are piled on top of each other in the order of increasing mesh sizes upwards as shown in Figure 3. After putting a certain amount of soil mass in the top sieve with the largest size, vibration is applied to the stacked sieves using water, if necessary to separate each grain. The weight of the soil retained in each sieve is measured and percentage of the soil remaining in each sieve is obtained. Based on this test data, the percentage of the soil having the particle size greater than 0.074mm, 0.105mm, 0.250mm etc. can be determined easily. The cumulative percentage value thus obtained is plotted versus the mesh size as illustrated in Figure 4. The mesh size is generally taken as indicative of the grain size. The curve thus established is called “Grain size distribution curve”. Thus, the point A3 in this figure indicates, for instance, that the 50% of this soil is composed of particles whose size is smaller than  $250 \mu$ . Notations of the grain sizes most frequently used in practice are illustrated in Figure 5 where  $D_{10}$ ,  $D_{30}$ , etc. implies the maximum size of the particles composing 10%, 30%, etc. of the total weight for the given soil. For example,  $D_{50} = 250 \mu$  means that 50% of the soil has particle sizes smaller than  $250 \mu$ . To represent the grain composition of a given soil, in a quantitative manner the following two, parameters are of frequent use.

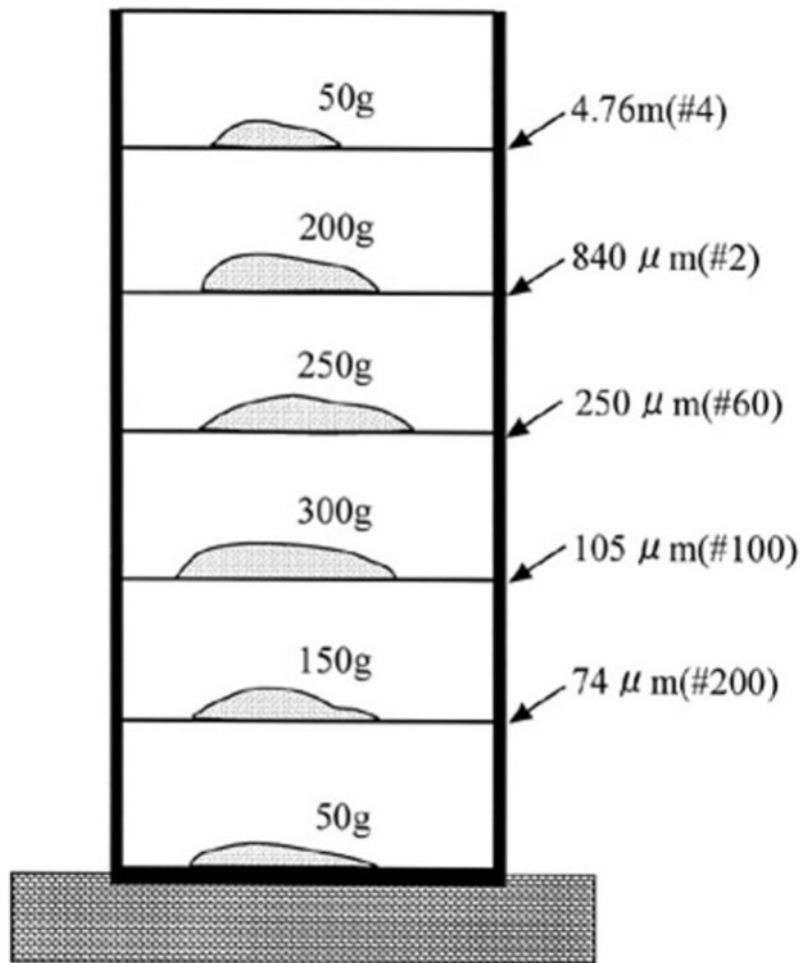


Figure 3: Test for sieve analysis

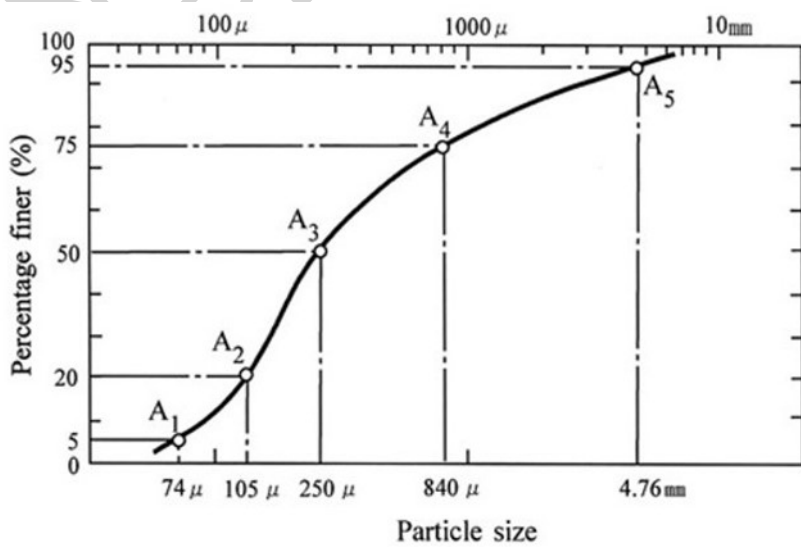


Figure 4: Method of plotting test data making up a grain size distribution curve



(a) Effective particle size  $D_{50}$

This parameter is used to indicate an average size of the entire particles composing a given soil. Sometimes, the particle size  $D_{60}$  is used as a parameter indicative of overall average size of the entire particles.

(b) Uniformity coefficient  $U_c$

The Uniformity coefficient is defined as  $D_{60}$  is used as a parameter indicative of overall average size of the entire particles.

$$U_c = \frac{D_{60}}{D_{10}} \quad (12)$$

The grain size distribution curves for two soils having different values of  $D_{60}$  but with an identical  $D_{60}$  are shown in Figure 6. Apparently, the soil A has the  $U_c$ -value which is greater than that of the soil B. Therefore, the uniformity coefficient indicates how widely or narrowly the grain size is distributed in the entire mass of a given soil. If the  $U_c$ -value is large, it implies that the soil comprises particles with widely ranging sizes and such a soil is termed "well-graded" whereas the soil comprising particles with almost uniform sizes is termed "poorly-graded".

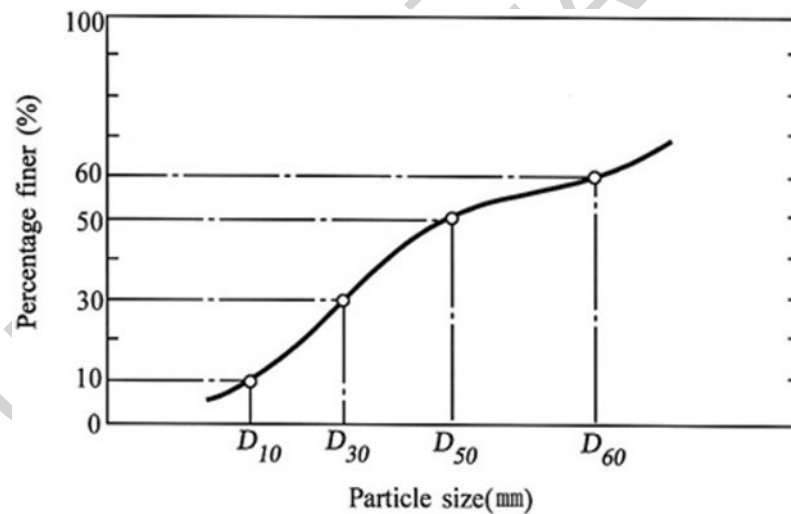


Figure 5: Grain size distribution curve

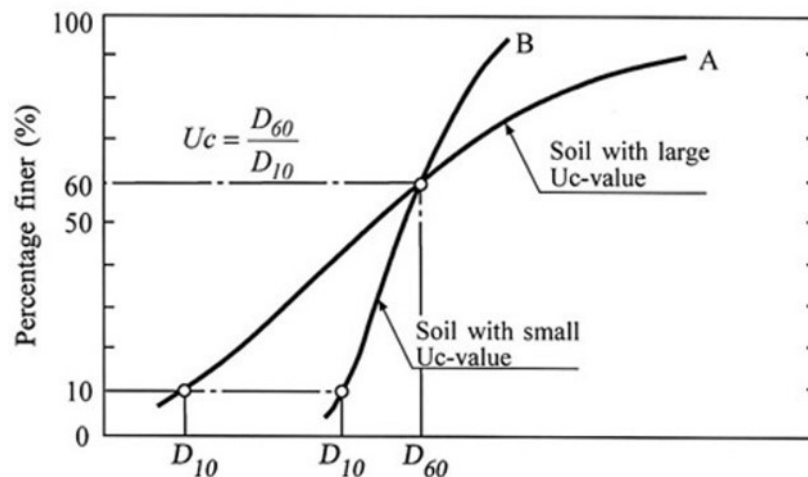


Figure 6: Definition of the uniformity coefficient,  $U_c$

## 2.4. Consistency of Fine-Grained Soils

The engineering properties of fine-grained soils can not be identified by the data on the grain size distribution curve and it is usual, instead, to perform what is called "consistency test". This consists of two types of simple tests, namely, liquid limit test and plasticity limit test.

### 2.4.1. Liquid Limit and Plastic Limit

It is well known that a soil becomes softer as the water content increases, leading to change in its behavior from solid to liquid. The threshold water content differentiating between the solid-like and liquid-like behavior is defined as the liquid limit. When water content decreases down below a certain value, the soil ceases to behave like a cohesive continuum leading to separation into many granules. The water content at this threshold condition is called the plastic limit. The liquid limit denoted by  $W_L$  or LL and the plastic limit by  $W_p$  or  $P_L$  are determined easily in the laboratory by means of simple tests of device.

First of all, sieve analysis is performed by using the No. 40 sieve to obtain fine-grained portion of a given soil for which the consistency tests are to be run. The fines are then mixed with water to obtain paste-like soil. Then, it is spread on the inside surface of the bowl-shaped container. A groove is created through the paste by a spoon-like cutting device. The paste-containing bowl is dropped many times until the groove is closed. This type of tests is repeated several times using soil pastes with different water content. By arranging the test data on a diagram, it becomes possible to determine the water content at which the groove is closed at a specified number of drops of the bowl. This is the liquid limit.

The plastic limit is determined by rolling the soil paste by palm until it becomes a thin thread with a diameter of 3mm. For varying water content, thread is separated into some small pieces is defined as the plastic limit.

### 2.4.2. Plasticity Index

The difference in water content between the liquid limit  $W_L$  and plastic limit  $W_P$  defines the plastic index which is denoted by  $I_p = W_L - W_P$  or PI. This index property indicates the range in water content within which the fine-grained soil can exhibit a sound and competent performance without being either fragmented due to lack of water or becoming liquid-like substance with too much water. Thus, the largeness or smallness of the plasticity index is indicative of the physical properties as follows.

(a) Plastic or cohesive soils have a higher value of  $I_p$  and can sustain large deformation without collapse and this competent behavior can be maintained within a wide range of water content. Heavy clay with considerable cohesion is an example of highly plastic clay. In contrast, fine-grained soils such as ground rocks have a low value of  $I_p$  and if saturated with water, it can easily fail by an external agitation and start to move significantly. Deposits of slurry from mine processing are a good example of the low plasticity soil and the tailings dams constructed of these materials are known to fail due to the agitation during earthquakes. When the plasticity limit test can not be conducted because of the lack of adhesion between particles, the soil is called non-plastic.

(b) Fine-grained soils with high plasticity index have a capacity to self-sustain as a continuum within a wide range of water content. Such soils have generally high water content and hence the void ratio is high. Therefore, if subjected to a compressive stress, it reduces its volume significantly. Thus, a soil with a high plasticity index is generally compressive. It is to be noticed that the high value of the plasticity index emerges from the fact that both the liquid limit and plastic limit take high values. Focusing on the high value of liquid limit, there are some empirical correlations proposed for the compressibility index  $C_c$  (Sec. 4.2) as a function of  $W_L$  as follows.

$$C_c = 0.009(W_L - 10) \quad (13)$$

### 3. Compaction of Soils

Compaction of soils is an important consideration in the design and practice of soil filling for construction of rock-earth fill dams, embankments for railways, highways and airfields. The mechanical properties of compacted soil fills are governed by the degree of compaction which is known to depend largely on energy of compaction and water content,  $w$ , at which the soil is compacted.

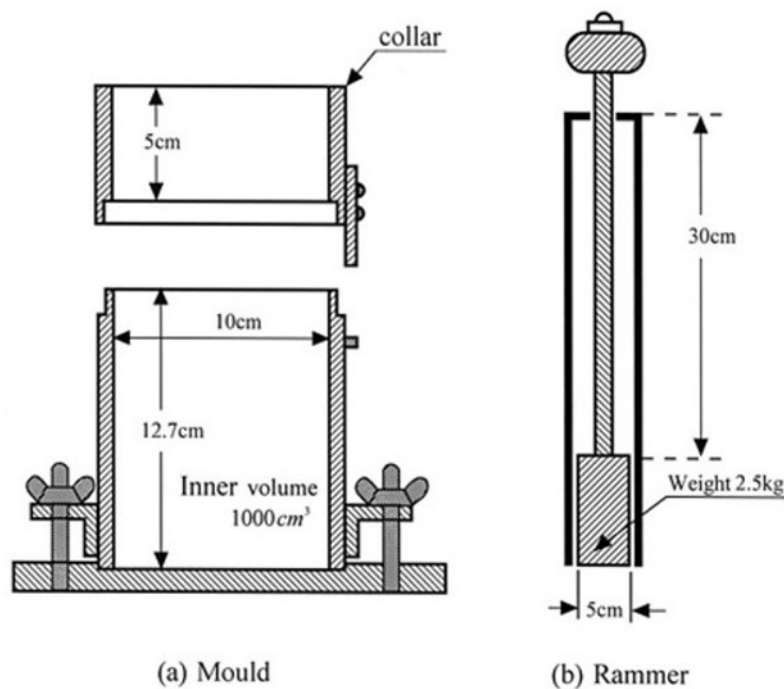


Figure 7: Device for standard Proctor compaction test

To specify the energy, the standard Procter test method was introduced in the ASTM code in U.S. and has been used to evaluate the compaction characteristics of a given soil. In this method a steel-made mould having the inner volume of  $V=1000$  cc as shown in Fig. 7 is used for testing the soil. The soil with given water content is placed in this mould in three layers and compacted each by dropping 2.5 kg-weight rammer 25 times from the height of 30 cm. This standardized method imparts a certain amount of specified energy to the soil tested in the mould. The compaction of the 1000 cc soil is repeated by stepwise changing water content,  $w$ , and dry unit weight,  $\gamma_d$ , is measured in each time of the test run. A set of data thus obtained is plotted in terms of the dry unit weight versus the water content as illustrated in Figure 8. It is well known that the dry unit of a soil compacted under an identical energy shows a maximum value  $\gamma_{dmax}$  at a certain water content which is termed optimum water content,  $\omega_{opt}$ , as illustrated in Figure 9. Thus, when soil filling is executed in the construction site, it becomes necessary to specify the water content at which compaction is to be made with maximum efficiency. Superimposed in Figure 8 a family of curves with different saturation ratios,  $S_r$ , as given by

$$\gamma_d = \frac{1}{1/G_s + W/S_r} \gamma_w \quad (14)$$

Thus, when the water content is smaller than  $W_{opt}$ , that is, on the dry side of the optimum, increasing dry unit weight is shown in Figure 8 to be accompanied by the significant increase in saturation ratio. This means that most of the compacting energy

is spent to reducing the air volume leading to the proportional increase in dry unit weight.

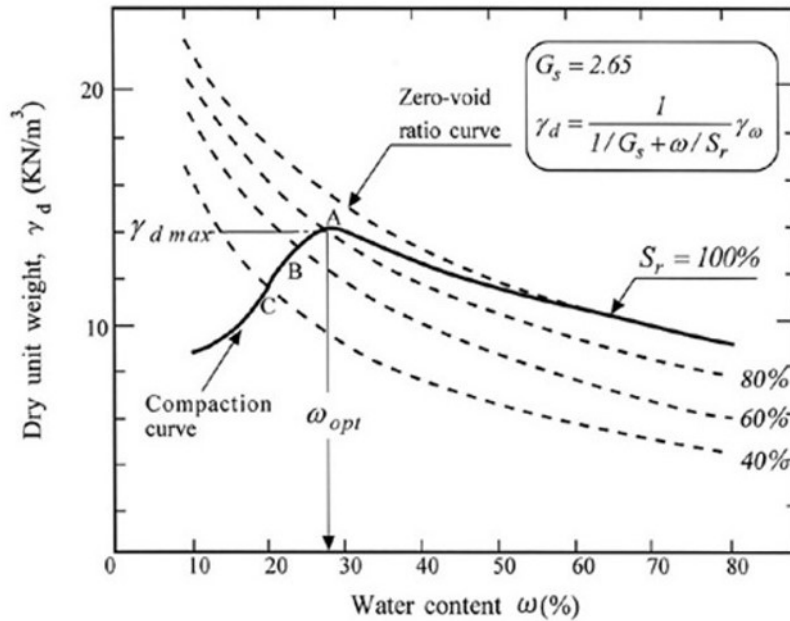


Figure 8: Compaction curve

When the water content is larger than  $W_{opt}$ , that is, on the wet side of the optimum, increase in water content does not produce much change in the saturation ratio. This fact indicates that the most of the compacting energy is spent to kneading or remolding of nearly saturated soil. The compaction curve tends to approach the curve of  $\gamma_d$  versus  $\omega$  with  $S_r = 100\%$  which is referred to as the zero-void ratio curve.

The compaction curve is dependent on the compacting energy which can be varied either by changing the number of hammer drop or the number of layering in the device of the standard Proctor test. If the compacting energy is increased, the compaction curve is shifted upwards thereby reducing the optimum water content and vice versa as illustrated in Figure 9. Therefore the optimum water content is uniquely determined only when the energy of compaction is specified.

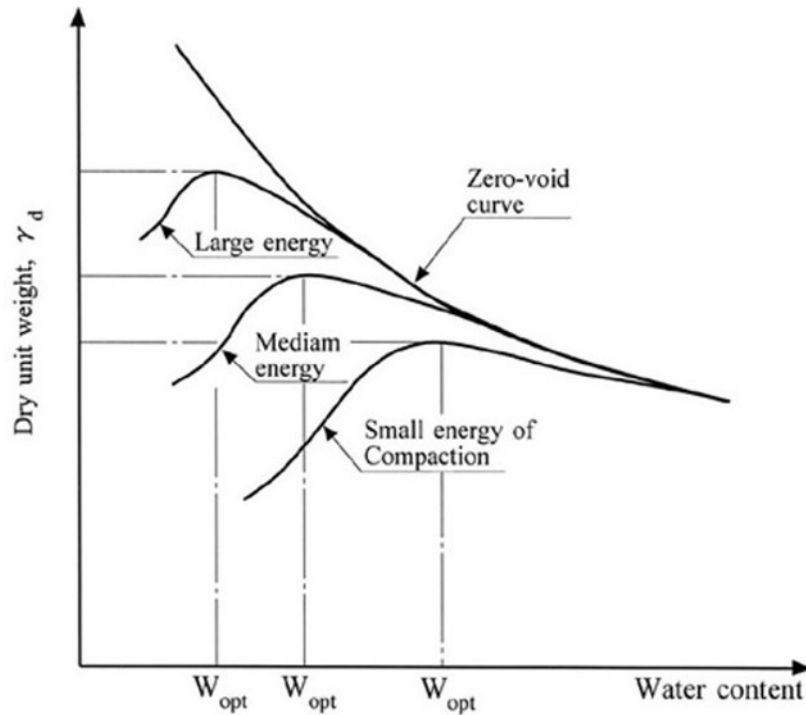


Figure 9: Effects of compacting energy on the compaction curve

It is known that the compaction characteristics are different depending upon soil type and grain composition of soils. For nine soils having the grain size distribution curves as shown in Figure 10, the compaction tests were performed employing an identical energy specified by the standard Proctor test. The results of the tests are presented in Figure 10 where it can be seen that the compaction curve tends to shift up leftwards with increasing grain size, indicating that the optimum water content decreases with increasing value of maximum dry unit weight attained.

In the field, different types of roller machines such as vibratory roller, sheep-foot roller or tamping roller are utilized to compact soils. In this case, pilot tests are generally conducted *in situ* to determine the number of the roller run, layer thickness and optimum water content for design purpose which is sufficient to achieve a targeted value of the dry unit weight.

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### **Biographical Sketch**

**Kenji Ishihara** had served as a professor of soil mechanics and geotechnical engineering at the University of Tokyo from 1977 to 1995 and then became a professor of civil engineering at the Science University of Tokyo from 1995 on to 2001. He now teaches soil mechanics and foundation engineering at Chuo University in Tokyo, Japan. He served as President of the International Society for Soil Mechanics and Geotechnical Engineering for the term 1997-2001.

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