ELECTRIC AND MAGNETIC FIELDS

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Summary

This chapter discusses historical development of electric field in theory and in practice. It deals with concepts such as electric charge, Gauss law, Lorentz force, Hall effects, Biot-Savart law, etc in detail and builds up complex fields described by Maxwell equations. All the information given here forms the basis of advanced electromagnetic theory.

1. Introduction

Modern society is electrified. We can hardly imagine what it will be like if suddenly we don't have electricity and electric devices. The subject of electric and magnetic fields, known as electromagnetism, is fundamental to electrical engineering. This provides the basic understanding of the principles of electricity and the whole spectrum of devices ranging from large-scale electric generators and motors, household appliances, to small wireless communication devices.

This chapter starts with historical development of the concept of electric field and then proceeds with the concept of magnetic fields that will lead to the Maxwell equations, which form the basis for more advanced electromagnetic theory. This chapter will serve as an introduction to subsequent chapters that follow in this topic level chapter.

2. Electrostatic Fields

2.2 Electric Charge

All matter, whether solid, liquid or gaseous, is made up of a number of atoms which themselves consist of a number of particles. There are many different subatomic particles, but the electron, the proton and the neutron are the most important of these.

An electron has a mass of 9.11×10^{-31} kg and carries negative electric charge. A proton has a mass of 1.6×10^{-27} kg and carries positive electric charge, the amount of which is exactly the same as the amount of negative charge of an electron. A neutron has the same mass as that of proton but has no electric charge. All atoms except the hydrogen atom have all three particles, and the protons and neutrons together form the nucleus of the atom. Since the total number of electrons equals the total number of protons, atoms are normally neutral electrically. When some of the electrons are removed from the atoms of a body, the body has a deficiency of negative charge. Since electrons cannot be destroyed some other body will have a surplus of electrons (i.e., charges are conserved). An atom that has lost electrons is called a positive ion, and one that has gained electrons is called a negative ion. A body that has a deficiency of electrons is said to be positively charged, while the body that has a surplus of electrons is said to be negatively charged. The smallest amount of charge is the charge of an electron which is 1.602×10^{-19} Coulomb (C), and all known particles have a charge that is an integer multiple of the fundamental charge $e = 1.602 \times 10^{-19}$ C, i.e., charge of particles are either 0, $\pm e$, $\pm 2e$, $\pm 3e$, $\pm 4e$, etc.

There is a force between any two point charges, which is experimentally determined first by Coulomb (1785). Coulomb showed that the force is proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them. This law is known as Coulomb’s law and stated mathematically as

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \tag{1}$$

where F in Newton (N) is the mutual force between the charges, Q_1 and Q_2 are the magnitudes of the two charges, r is the distance between them, and ε is the permittivity of the medium.

The force acts along the line joining the points and is in such a way that the like charges repel and the unlike charges attract to each other. For bodies with charges, it is difficult to define the distance between them. Assuming that the distance between the charges is large compared with radii of the charges, the distance between them can be taken to be the distance between their centers.

The permittivity constant ε in a vacuum is called the absolute permittivity or the primary electric constant $\varepsilon_0 = 8.854 \times 10^{-12}$ measured $C^2N^{-1}m^{-2}$ in SI units. The same value is normally assumed for air since its value is only 0.06% less than the value for vacuum.

The Coulomb's law is of course valid in a microscopic level for an atom, in which case the charges are those for an electron and a proton, i.e., having magnitudes of 1.602×10^{-19} C. The *electric or electrostatic force* holds the electrons near the nucleus. Notice how an atom resembles the solar system. In a solar system, the *gravitational force* is responsible for holding the orbit of planets around the sun, where the gravitational force is expressed by

$$F = \frac{GM_1M_2}{r^2} \qquad (2)$$

where M_1 and M_2 are the masses of two objects, r is the distance between them, and G is the gravitational constant. For an atom, the electrostatic force between an electron and a proton is 10^{40} times larger than the gravitational force between the two particles. Therefore, in calculating the force between charges, the gravitational forces are neglected. All contact forces of everyday experience (e.g., the push of your hand on a door) are simply combined effect of electric forces of atoms. You have never really touched anything in your life! What really happens when we study the "direct action" of one piece of matter right against another? We discover that it is not one piece right against the other; they are slightly separated, and there are electrical forces acting on a tiny scale (Feynman, Leighton and Sands, 1964).

Electric charge on a macroscopic level is the result of the accumulation of large numbers of atomic charges each having magnitudes of 1.602×10^{-19} C. These charges may be positive or negative, protons being positively charges and electrons negatively. Among the particles, only electrons are movable charges while the protons remain fixed in nearly all electronics circuits. The exceptions to this are when conductions are taking place in liquids and gases, where positive ions may contribute to the electric current.

2.2 Electric Field

An alternative method of computing the electric force is the use of *electric field strength.* The electric field strength at a point is the electric force that would be experienced by a unit positive charge (i.e., a charge of 1 C) placed at the point. In the Coulomb's law, Eq. (1), the electric force can be re-written as

$$F = Q_1E \qquad (3)$$

$$E = \frac{Q_2}{4\pi\varepsilon r^2} \qquad (4)$$

where E is defined as the electric field strength, measured in Newton per coulomb (NC^{-1})in SI units. The step of introducing E is important because it separates the source of the electric force (Q_2) from its effect on the charge Q_1.

The region in which forces are experienced due to the presence of electric charges is called an *electric field*. At each point within this region the electric field strength will have both a magnitude and a direction, and the electric field of the region is often represented by diagrams like Fig. 1 in which the lines, referred to as lines of force, show the direction of E. The arrowheads indicate the direction of the force that would act on a positive charge placed in the fields. The spacing of the lines of force is inversely proportional to the magnitude of the field.

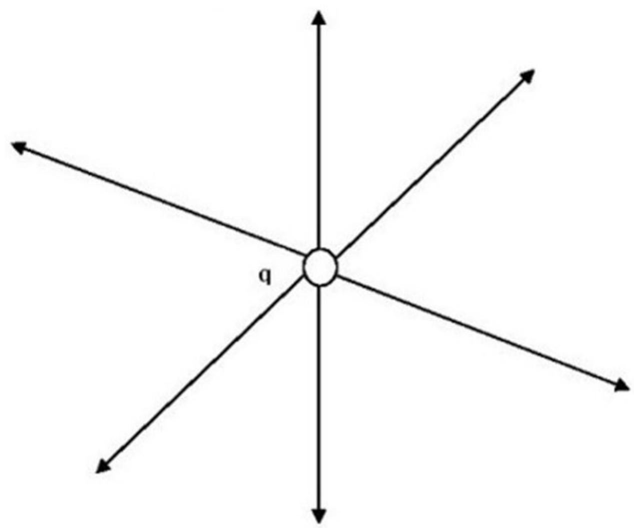

Figure 1: The lines of force of the electric field of a point charge emanates from the positive charge in all directions

When there are many charges in a region, the force acting on a charge due to the whole assembly of other charges can be determined by the principle of superposition. This can be seen from the definition of the electric field strength Eq. (4), where the electric field strength is proportional to the charge producing it. Therefore, the electric field produced by an assembly of charges is the vector sum of the fields due to individual charges. The principle of superposition is valid, however, only for linear systems. The response of some materials to electric fields is non-linear and the use of the principle is not valid in these materials. In this case, the permittivity in Eq. (4) is no longer constant, but changes as a function of the electric field strength.

2.3. Gauss' Law

Figure 1 shows electric field lines radiating from a charge. The total electric flux coming out of the point charge can be computed by enclosing the charge with a closed surface. The simplest choice of the enclosure is a Gaussian surface, which is a sphere concentric with the charge. Electric field lines or E is always normal to the surface of the sphere and its magnitude is constant there. In this case, total flux of *E* out of the sphere is the product of the magnitude of *E* with the surface area of the sphere:

$$\psi = E \cdot 4\pi r^2 = \frac{Q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{Q}{\varepsilon_0} \qquad (5)$$

where ψ is the flux of *E*. Note that the flux of *E* out of the sphere is independent of the radius of the sphere and depends only on the charge enclosed within it. It is true for any shape of surface and, by using the superposition principle, for any number of charges enclosed. This is the Gauss' law and may be stated as follows:

> *The net flux of E in free space out of any closed surface is equal to the charge enclosed by the surface divided by* ε_0.

This can also be written mathematically as

$$\oint_S \underline{E} \cdot \underline{dA} = \frac{Q_{\text{net}}}{\varepsilon_0} \qquad (6)$$

where S is a closed surface enclosing the net charge Q_{net}, arrows indicate the quantities are vectors, meaning they have magnitudes and directions, and $\underline{E} \cdot \underline{dA}$ is electric flux through an area $\underline{dA}$.

For problems involving dielectric materials, it is more convenient to use a new vector known as the electric flux density *D* defined by

$$D = \varepsilon E$$

Then the Gauss' law, Eq. (6), becomes

$$\oint_S \underline{D} \cdot \underline{dA} = Q_{\text{net}}$$

or, in a general form,

$$\oint_S \underline{D} \cdot \underline{dA} = \int_V \rho dv \qquad (7)$$

where *S* is the surface enclosing the volume *V* and ρ is the charge density. Gauss' law can be used to determine electric field due *E* to a charge distribution over an area or volume.

Point charge:

Consider a point charge enclosed by an imaginary closed surface. It is convenient to choose a spherical surface, because in this case, the electric field vector will always be perpendicular to the surface. Fig. 2 shows a three-dimensional view of the closed surface, which is a spherical shell of radius *r*

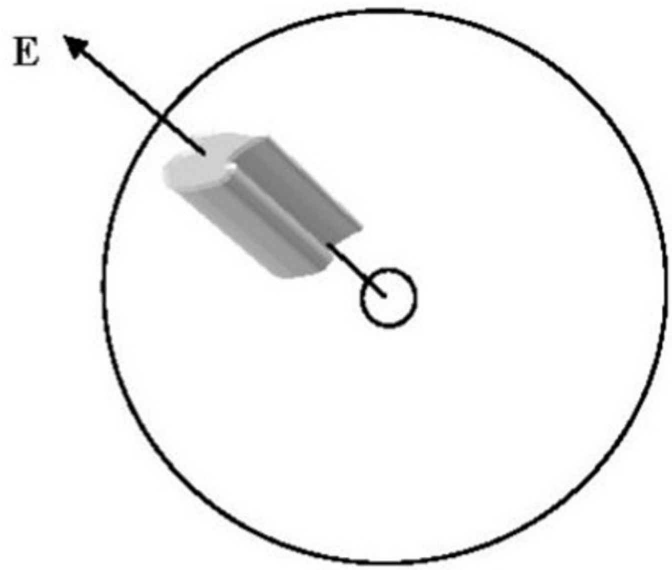

Figure 2: A spherical shell enclosing a point charge

From Gauss's law

$$\oint_S \underline{E} \cdot \underline{dA} = \frac{Q_{\text{net}}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \quad (8)$$

The electric field *E* going through any element on the sphere is the same, because in each case, the distance of the element from the point charge *Q* is the radius *r*.

$$\oint_S E \cdot dA = E\oint_S dA = \frac{Q}{\varepsilon_0} \quad (9)$$

Since the integral is the addition of all small areas on the surface, it is simply the surface area of the sphere, $4\pi r^2$. Thus Eq. (9) becomes

$$E 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad (10)$$

and hence, the electric field is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad (11)$$

which agrees with the electric field defined in Eq. (4).

Charged rod:

Figure 3 shows a cylinder having charge *q* per unit length distributed uniformly over its surface.

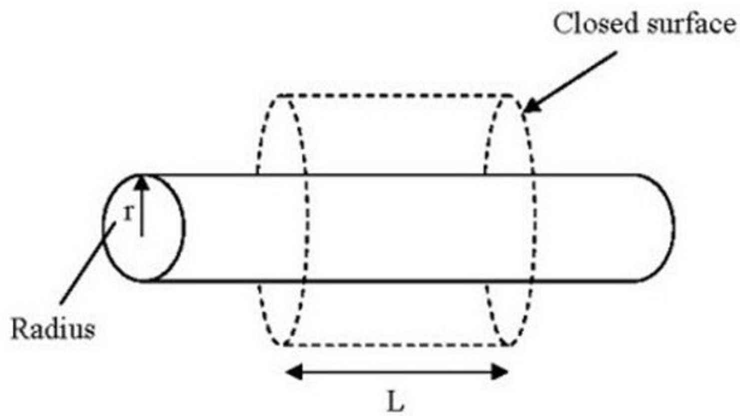

Figure 3: A Gaussian surface for calculating the electric field strength around a charged cylindrical rod

From the Gauss' law

$$\oint_S \underline{E} \cdot \underline{dA} = \frac{Q_{\text{net}}}{\varepsilon_0}$$

by assuming that the field strength *E* is directly radially outwards and the magnitude of *E* depends only on the distance from the axis, for the length *L*

$$E 2\pi r L = \frac{qL}{\varepsilon_0}$$

or

$$E = \frac{q}{2\pi\varepsilon_0 r} \qquad (12)$$

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Biographical Sketch

Professor Kwang Y. Lee, Director of Power Systems Control Laboratory, has been an educator and researcher in power systems and control systems, especially in intelligent control systems. He has authored and co-authored over 350 papers and 19 book articles in the area.

Dr. Lee's interest has been in the area of power systems control. In power systems research, he has contributed methodologies for optimal long-term generation and transmission expansion planning, optimal power system operation, and power plant control.

Dr. Lee's strength has been in the blending of modern control theory into power systems study. His active participation in teaching and research in control systems has helped him in developing advanced technologies for power systems and power plant control. Besides modern control theory, the latest developments in artificial intelligence, robust control, and nonlinear mathematics are important tools for him in solving power systems problems.

To promote international recognition, Dr. Lee coordinated the U.S.A.-Korea Joint Seminar on Expert Systems for Electric Power Systems with the support of NSF and the Korea Science and Engineering Foundation (KOSEF).

Professor Lee is an IEEE Fellow and is also an Associate Editor of the IEEE Transactions on Neural Networks and Editor of the IEEE Transactions on Energy Conversion.