SIGNALS AND SYSTEMS

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Contents

Summary

The chapter *Signals and Systems* is concerned with the representation of signals and the study of the changes that occur to them as they pass through systems. The study of Signals and Systems is, by necessity, highly mathematical since its focus is to allow one to predict, with some certainty, the behavior of systems when they are subjected to different input signals. The mathematical approach also allows one to design electrical

circuits or algorithms that will operate on signals in such a way so as to produce desirable outcomes.

Signals and Systems find many applications in a diverse range of disciplines. This lead chapter in the theme provides an overview of the concept, the theory, and the mathematical tools available to the practitioner in the field. Greater details of the methods and in depth knowledge about the Signals and Systems are given in: *Signal Theory, Digital Signal Processing, Analog Signal Processing, Image Processing, and Modulation and Detection.*

In this chapter, we present an overview of the topic of Signals and Systems, focusing on the key concepts of the topic. We first provide the motivation for adopting the mathematical approach and discuss some practical limitations to this approach and the compromises that one must accept. Following this discussion, we summarize the main points of signal theory, in particular, the different representations of signals and the relationships between these representations. The main points of systems theory, especially those relating to the important class of linear time-invariant systems, are next discussed.

This chapter concludes by presenting two applications to highlight the usefulness of the concepts in Signals and Systems. The first example comes from feedback control and shows how one can stabilize an unstable system. The second example comes from adaptive signal processing and shows how one can use noise canceling techniques to extract a signal buried deep in noise.

1. Introduction

The importance of *Signals and Systems* in electrical engineering can be seen readily by examining the curriculum of a typical undergraduate electrical engineering course. Invariably, one will find that *Signals and Systems* is taught as a core subject. The reason for this is because the concepts of Signals and Systems form the foundation upon which the theories and techniques of many later year subjects that are studied in electrical engineering are developed. These subjects include electrical and electronic circuit design, photonics, electromagnetics, signal processing, telecommunication systems, control systems, electrical machines, and electrical power systems. The importance of Signals and Systems is not restricted only to electrical engineering but it is as applicable to all other engineering disciplines, and indeed, in other seemingly unrelated fields such as seismology, economics, sociology, transportation, public and private administration, and political systems.

From a different perspective, Signals and Systems also have an interesting role to play in the personal development of an engineering student. The problem solving and analytical skills that the student acquires through the study of the subject can often help him in his later career to tackle and solve a myriad of problems in a systematic and cogent manner.

Looking ahead, the scope of potential applications of the methods of Signals and Systems will continue to grow as engineers are faced with new challenges involving ever more complex systems and processes.

2. The Elements of Signals and Systems

In the study of *Signals and Systems*, although the nature of the problems may differ greatly from one application to another, they all have two common elements. Firstly, there exist signals which are simply phenomena that can be described quantitatively and contain information about the behavior or nature of the system under study; and secondly, we are only interested in the *input-output* behavior of the system, that is, its response to *input signals*, where the response may take the form of other signals being produced by the system, called *output signals*, or of the system exhibiting some desired form of behavior. This input-output view of Signals and Systems is represented pictorially in Figure 1. Some examples of systems, and their associated signals, are described below.

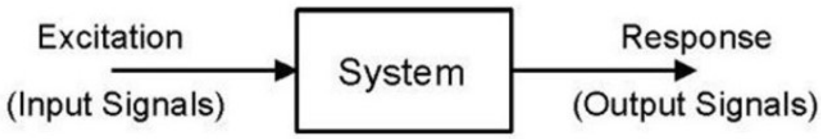

Figure 1. Pictorial representation of a system and its associated signals

An example of signals and systems in electrical engineering is the radio receiver circuit where the input signal is the electromagnetic wave that emanates from a broadcaster's station picked up by an antenna. The output signal is the voltage which then applied to an audio amplifier–loudspeaker combination to produce the sound that one hears. Another example is a hair dryer. The input signal in this case is the current drawn from the power mains and the output signal is the heated airflow. It is interesting that in both examples, there are further input signals. These are contributed by the user of the equipment. For the radio receiver, this is the setting on the volume control while in the hair dryer, it is the setting of the heater and fan speed control.

For an example in a non-engineering field, consider the national economy of a country. The signals of this example will include the national income and expenditure, its labor force and natural resources, and infrastructure.

Finally, we point out that the system under study does not have to be a physical entity. It can be notional. An example of such a system is an algorithm or formula, to calculate our income tax. The taxation formula accepts as input signals a set of numbers – our earnings and allowable deductions – and produces as output signals, another set of numbers – our income tax. Although the taxation formula is non-physical, its output signal, that is, the tax we pay, is undeniably physical.

3. The Mathematical Approach to Signals and Systems

Recall that the focus in the study of Signals and Systems is on the input-output behavior of a system. More precisely, we are interested in being able to predict, quantitatively, the output signals of a system, given its input signals. This implies that one must be able to characterize the input-output behavior of a system by means of a *mathematical* relationship, and to represent its input and output signals by some mathematical function. It is in this abstract mathematical framework that makes the concepts and technique of Signals and Systems equally applicable to a field of diverse applications.

The mathematical approach to the study of signals and systems is commensurate too with the ultimate goal of engineering, that is, to design, build and operate, at an acceptable financial, environmental and social cost, entities that will perform some beneficial desired task(s). The mathematical approach allows the design engineer to weigh up various design options before proceeding to the building stage. In many instances, it is this very ability to predict the performance of the final design that is crucial. For example, to land man on the moon, or to build a nuclear power plant, one basically has to get it right on the first attempt. Failure can be disastrous.

From the above discussion, it is clear that a critical requirement of the design process is that one has a *mathematical model* of the system being designed. Indeed, the topic of mathematical modeling is very broad and warrants a separate treatment. From the standpoint of engineering design, however, useful mathematical models are those that represent a good compromise between analytical simplicity and modeling complexity. That is, they are simple enough to allow the designer to readily extract from them information about the system being designed, but at the same time, they are complex enough that they can reveal the main characteristics of the system. The challenge is to derive suitable mathematical models that are appropriate to the design task at hand.

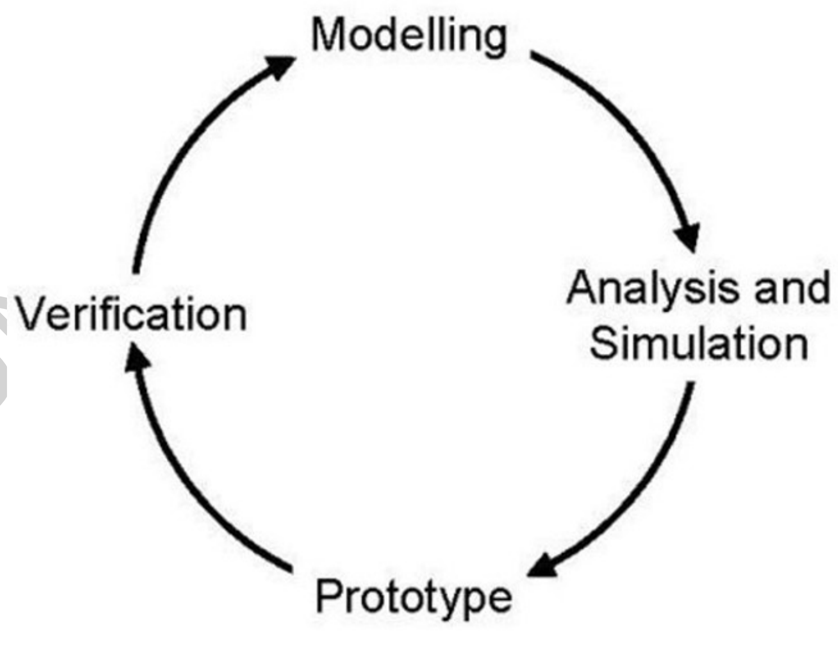

Figure 2. The engineering design cycle

Indeed, the process of modeling is often integrated into the design process itself as illustrated in Figure 2. Note that the design process is actually a cycle of events. One starts the design cycle by constructing a model of the system. With this model, one then performs a (conceptual) design and determines its performance either by analysis or simulation, or a mixture of both. Once the design is deemed satisfactory, one builds a prototype. This may involve its own design cycle. The performance of the prototype is then verified, or tested, in the sense that one subjects the prototype to certain test signals and measures its response. This may involve another design cycle as one may need to work out novel ways of applying the test signals and measuring the response. Invariably, there will be a gap between the expected response and the actual response. This gap may be reduced by making some minor adjustments to the prototype. If the final gap is within the design specifications, then the design task is completed and one moves to the final construction phase. If, however, the gap is still too wide, then one will have to review the whole design cycle to find out the cause of the gap. This may include a revision of the mathematical model. The insights gained from the verification stage can be helpful in refining the model.

4. Signal Representation

In Signals and Systems, although all signals are ultimately abstracted to nothing more than just pure mathematical functions, there still remains the issue of how to represent these functions. Which representation to use will depend on the system under study. In this section, we outline a number of ways to represent signals.

4.1 Continuous-Time and Discrete-Time Signals

In electrical engineering, most naturally occurring signals are continuous-time in nature. That is, they can be represented by the function $x(t)$ where x denotes the signal and t, the time variable, is defined for all reals, i.e., $t \in \mathbb{R}$. Examples of continuous-time signals include the radio signal emitted from a cellphone, and the output voltage of a microphone as it converts the sound pressure wave it senses to an electrical signal.

The microphone example above highlights an important point in electrical engineering. That is, the importance of *sensors* and *actuators*. A sensor is a device that gives an electrical signal (voltage or current) for the detection or measurement of a physical property to which it responds, while a actuator performs the reverse operation, that is, it produces a physical action that is directly proportional to the electrical input it receives. Thus a microphone is a sensor, while a loudspeaker is an actuator. Once a physical quantity has been converted to an electrical form by a sensor, then all manner of electrical and/or electronic operations can be performed on the signal. The result of operations is then interfaced back to the physical world through an actuator.

In the case of discrete-time signals, these signals are defined only at certain instants of time. Often, these time instants are uniformly spaced. Accordingly, a discrete-time signal can be represented by the function $x(nT_s)$ where T_s is the time interval between the instants at which the discrete-time signal is defined, and n is an integer, i.e., $n \in \mathbb{Z}$. For convenience, we often drop the reference to T_s and denote the discrete-time signal by $x[n]$. An example of a discrete-time signal is our bank balance at the end of each day when our interest is calculated. With modern technology, there is now a strong trend towards operating on continuous-time signals by means of a digital computational element (see *Digital Signal Processing*) such as a general purpose digital signal processor (DSP). The DSP approach has many advantages such as flexibility in the ease in changing the operation of the DSP, repeatability in that repeating the same operation on the same input signal will produce the same result, and reproducibility in that running the same algorithm on a different DSP will also produce the same result. For faster computational throughput, dedicated devices such as a custom designed very large scale integrated circuit (VLSI) or a field programmable gate array (FPGA) are also used. The point here is that, by nature digital computational elements operate in discrete-time, at a rate determined by their internal clocks. The continuous-time signal must thus be *sampled* by an *analogue-to-digital converter* (ADC) which converts the continuous-time signal to a discrete-time signal. The reverse operation, that is, the conversion of the computer's discrete-time output signal to a continuous-time signal, is performed by a *digital-to-analogue converter* (DAC). Typically, signal conversion occurs at a fixed rate called the *sampling rate*. The sampling rate is given by $1/T_s$ where T_s is the *sampling period*.

Despite the current preponderance towards digital signal processing, the importance of analogue signal processing (see *Analogue Signal Processing*) has not diminished. The ability of a digital signal processing system to perform a certain signal processing task is limited by the computational capacity of its central processing unit (CPU). Often, it is not realistic to implement highly involved signal processing functions using digital signal processing, and the only practical solution is to use analogue signal processing. Also, digital signal processing requires the input continuous-time signal be conditioned in some sense (see Section 4.3). Signal conditioning is in the domain of analogue signal processing.

4.2 Continuous-Amplitude and Discrete-Amplitude Signals

The amplitude of most signals that arise in engineering can take on a continuum of values. However, occasionally, we encounter signals whose amplitude can take on only discrete values. Examples of such discrete-amplitude signals include the progressive score in a game of football or the Dow Jones Index in the stock market.

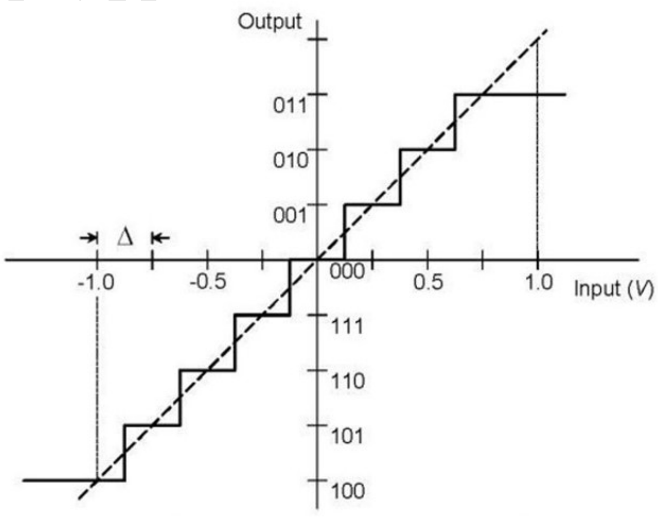

Figure 3. Input-output characteristic of an ideal 3-bit ADC with $\Delta = 2/2^3 = 0.25$ V and

digital output in 2's complement format.

The signals that are represented inside a digital computer after they have been converted by an ADC are also discrete in amplitude. This is due to the construction of the ADC. A characteristic of ADCs is their *quantization step*. Suppose the quantization step of an ADC is Δ. If the amplitude of the ADC input signal, at the nth sampling instant, satisfies $(k - \frac{1}{2})\Delta \le x[n] \le (k + \frac{1}{2})\Delta$, then the ADC will output the integer k in binary. For a K-bit ADC, k will be limited to between -2^{K-1} and $+(2^{K-1} - 1)$. See Figure 3. Note that the possible amplitudes of the quantized signal are uniformly spaced, as illustrated in the figure.

In Figure 4, we depict the four possible combinations of continuous-time and discrete-time, and continuous-amplitude and discrete-amplitude signals. We also define in this figure what is meant by an *analogue signal* and a *digital signal*. As indicated, a digital signal is, strictly speaking, discrete in amplitude and time. However, for the sake of mathematical tractability in the study of discrete-time signals and systems, the amplitude of a digital signal is often taken to be continuous. Thus, by "digital signal processing", one often actually meant "discrete-time signal processing".

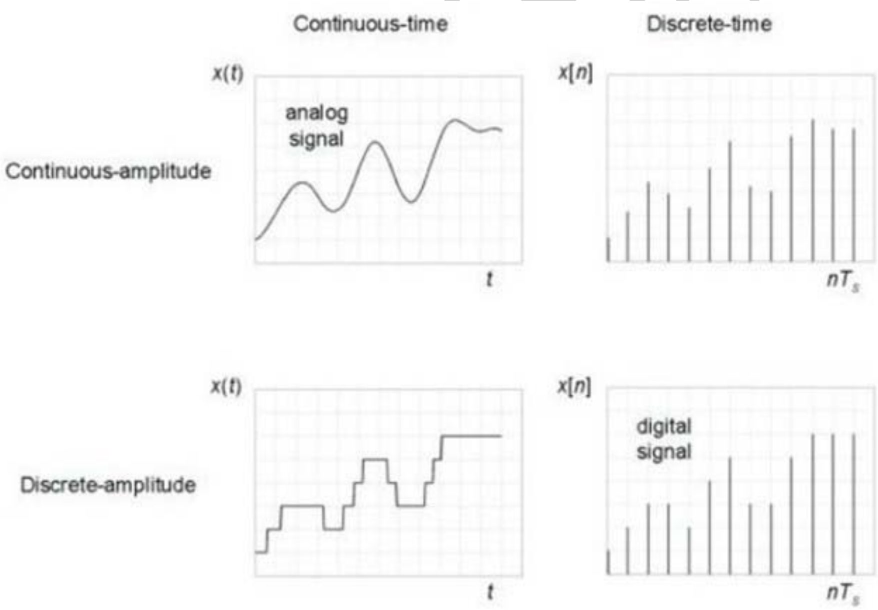

Figure 4. Classification of signals.

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Biographical Sketches

Yee Hong Leung obtained the BE (Hons) and PhD degrees, both in electrical engineering, from the University of Western Australia in 1977 and 1986, respectively. He has worked with the School of Electrical, Electronic and Computer Engineering, the University of Western Australia, as a Tutor from 1982 to 1986 and as a Lecturer from 1988 to 1994. During 1986 to 1988, he was a Research Scientist with the Defence Science and Technology Organisation, South Australia. In 1994, Dr Leung joined the Western Australian Telecommunications Research Institute, Curtin University of Technology, Australia, as a Senior Research Fellow. He is also associated with the Australian Telecommunications Cooperative Research Centre. He is now a Senior Lecturer in the Department of Electrical and Computer Engineering, Curtin University of Technology. Dr Leung's research interests are in adaptive and optimum broadband and narrowband antenna array signal processing, and in particular, applications of antenna arrays in telecommunication systems

Buon Kiong Lau received the BE(Hons) and Ph.D degrees in electrical engineering from the University of Western Australia and Curtin University of Technology, Australia, in 1998 and 2003, respectively. During 2000-2001, he took a year off from his Ph.D studies to work as a Research Engineer at Ericsson Research in Kista, Sweden. From 2003 to 2004, he was a Guest Research Fellow at the Department of Signal Processing, Blekinge Institute of Technology, Sweden. He now holds a joint Research Fellow appointment at the Radio Systems group and the Electromagnetic Theory group in the Department of Electroscience, Lund University, Sweden. During 2005, he was also a Visiting Researcher at the Laboratory of Information and Decision Systems, Massachusetts Institute of Technology, USA. Dr Lau's research interests include array signal processing, wireless communication systems, and antennas and propagation.