

# PERFORMANCE EVALUATION OF WATER RESOURCES SYSTEMS

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## Summary

A discrete-time dynamic system framework is provided for a coherent evaluation of water resources planning and operation. The six components of the model are:

- (a) the time scale
- (b) the state set
- (c) the input set including controllable and non-controllable factors as well as performance indices (criteria)
- (d) the output set, including figures of merit (super-criteria)
- (e) the state transition or system function
- (f) the output function

Emphasis is placed on risk and reliability criteria, and their trade-off. This last step leads in a natural fashion to a multiobjective approach.

## 1. Introduction

The purpose of this section is:

- to provide a system framework for performance evaluation of water resources system with emphasis on criteria related to risk and reliability in hydrology, and
- to illustrate the concepts by means of realistic examples.

Thus, before defining the criteria that may be used to evaluate water resources systems, a system framework is defined. Note that this section does not dwell very much on standard economic evaluation criteria, such as benefit-cost or benefit-risk because those are presented elsewhere in this volume, as well as in standard books.

An incident or failure event may occur in one of two ways:

- (a) A structure such as a spillway or a piece of equipment such as a pump fails; this is irreversible, and the item has to be rebuilt or replaced.
- (b) A resource such as irrigation water or hydropower is not supplied for a certain time when in demand, but then service is restored. Such events of varying severity, duration and magnitude may occur several times during the lifetime of the system.

For a long time these two cases of incident or failure have been studied separately using different criteria. In case (a) the system reliability is measured in terms of the time elapsed until the first (complete) failure or the system lifetime. In case (b), one considers first, the percentage of time that the system has performed satisfactorily during its lifetime, and then how well it has performed. We provide herein a system model that covers both cases. The elements of this model are:

- (a) A generalized load,  $L$ , which may be the hydrostatic pressure on an earth dam or the nitrate loading into a river.
- (b) A generalized resistance,  $R$ , which may be the weight and compactness of the dam, and the self-cleaning capacity of the river.
- (c) An incident or failure mode,  $m$ , which describes the mechanism of the event, such as slippage of the dam or extreme low flow of the river.

These three elements are combined into the definition:

$$\text{Incident } m \text{ occurs if, and only if, } L > R \quad (1)$$

As another example, consider in the case of water being supplied from sources or storage to satisfy users' demand, the load  $L$  would be the total demand and the resistance  $R$  would be supply capacity. Other possible interpretations for  $L$  and  $R$  are available.

## 2. System Framework

### 2.1 System Elements

Consider a discrete-time dynamic system  $Z$ , as defined by Wymore. For the sake of convenience, let this system be also discrete-state. Consider a single multi-purpose reservoir for the sake of simplicity. The system components are (T,S,X,Y,F,G).

T: a time scale ( $j = 1, 2, \dots, t, \dots, J$ ); the horizon  $J$  may be infinite.

S: a discrete (possibly infinite) state set for describing the system at time  $j$ . The state vector  $\mathbf{s}(j)$  belongs to set  $S$  and may include storage volume, water quality indices, cumulative shortage and flood volumes as well as physical characteristics of a dam,

such as resistance or fatigue. Let this state vector  $\mathbf{s}(j)$  also include performance indices  $PI^k$ ,  $k = 1, \dots, K$ , which measure the performance of the system with respect to a set of objectives, goals, purposes or targets. Let  $\mathbf{PI}(j)$  be the performance index vector at time  $j$ . The state trajectory at time  $t$  is the matrix of vectors:

$$[\mathbf{S}(t)] = (s(0), s(1), \dots, s(t)).$$

The performance trajectory is defined as the matrix:

$$[\mathbf{PI}(t)] = (PI(1), \dots, PI(t)), \text{ with } PI(j) \in \mathcal{S}(j) \quad (2)$$

$X = (U, W)$  is an input set composed of a set of controllable elements  $U$  and a set of non-controllable elements  $W$ . Controllable or decision vector elements  $\mathbf{u}(j) \in U$  may include both those parameters which are fixed in the planning process, such as the height or the site of a dam, and adjustable quantities, such as the opening of a spillway gate. Non-controllable elements  $w(j) \in W$  include such quantities as the total net inflow into a reservoir or the aging of a structure. A particular subset of  $U$  is the decision trajectory or control policy and a subset of  $W$  may be the random inflow sequence.

$Y$  is an output set comprising measured and calculated variables, such as water volume delivered, generalized loads  $L$  and losses or benefits. At every time  $j$ , the output usually includes an updated value of performance indices  $PI(j)$ , which were defined as elements of the state set  $\mathcal{S}$ , such as benefits from hydropower or losses due to excess spills.

The output also includes so-called figures of merit (FM), which are criterion functions defined over the components of the performance matrix  $[\mathbf{PI}(t)]$  (see Eq. (2)) where  $t$  is the time horizon; for example, usual FMs may be the mean value, the maximum or minimum component of  $[\mathbf{PI}(t)]$ . Generally speaking, the outputs  $y(j) \in Y$ ,  $j = 0, \dots, t$ , define the output trajectory:

$$[\mathbf{Y}(t)] = (y(1), \dots, y(t)),$$

a subset of which may be the performance index trajectory  $[\mathbf{PI}(t)]$ .

$F$  is a state transition function which calculates the state at time  $j + 1$  as a function of the previous states  $s(j)$  and the previous input  $x(j)$ :

$$s(j + 1) = F(s(j), x(j)) \quad (3)$$

Note that this system model possesses memory because by recurrence,  $s(j + 1)$  depends on all past states and inputs; also function  $F$  in Eq. (3) includes the algorithm to calculate the performance index vector  $\mathbf{PI}(j + 1)$ .

For example, in the case of reservoir operation, one component of vector function  $F$  is often the mass balance equation:

$$v(j+1) = v(j) - u(j) + w(j) \quad (4)$$

where

$v(j)$  = water volume in storage at time  $j$

$u(j)$  = controlled outflow, including release

$w(j)$  = net random inflow

Other components of  $F$  may include design parameters, sediment movement equations or trophic state evolution. An example of calculation of an economic performance index by means of Eq. (3) is as follows:

Consider, as an element of  $F(\cdot)$ , the rule to calculate the benefit function  $B(j)$  or the loss function  $C(j)$  at time  $j$ . Note that both  $B(j)$  and  $C(j)$  depend, in general, on storage volume  $v(j)$ , release  $r(j)$  and demand  $D(j)$ . A loss function under demand  $D(j)$  may be:

$$C(j + 1) = \begin{cases} r(j) - D(j) & \text{if } D(j) < r(j) < v(j) \\ a(D(j) - r(j)) & \text{if } r(j) < D(j) \text{ with } r(j) < v(j), a > 0 \end{cases}$$

This equation is a particular case of Eq. (3) with state variable  $s(j) = v(j)$ , and input  $x(j) = [r(j), D(j)]$ . The non controllable input is the demand  $w(j) = D(j)$ .

$G$  is an output function which provides the algorithm for calculating the present output  $y(j)$  as a function of the present state  $x(j)$ :

$$y(j) = G(s(j)) \tag{5}$$

A simple example of output function is the transformation of a shortage incident into an economic loss.

## 2.2 System Simulation or Experiment

To operate, for example, a water supply system, the input vector or trajectory is manipulated to provide a desirable output vector or trajectory. The desirability of an output is measured by means of PIs, calculated on the basis of a system simulation run, also called system experiment  $e(t) = (x(t), s(o), t)$ , composed of an input trajectory (inflow)  $x(t)$ , an initial state (storage)  $s(o)$ , and a time  $t$ . The state transition function  $F(\cdot)$  and the output function  $G(\cdot)$  determine, respectively, the state trajectory  $[S(t)]$  and the output trajectory  $[Y(t)]$ , given any experiment  $e(t)$ , as described in Table 1. Such a table may be used to describe the results of any digital simulation run, with decision trajectory  $\mathbf{u} = [U(t)]$  and non-controllable input trajectory  $\mathbf{w} = [W(t)]$ .

Although in principle a performance index PI, such as grade of service (defined later), may be calculated at every time  $j$ , one usually only evaluates a single performance index value  $PI(t)$  per system experiment and then one figure of merit value per ensemble of experiments. The figure of merit thus appears to be a “super-criterion.”

Time $j \in J$	Input $\mathbf{x}=(\mathbf{u}, \mathbf{w})$	State $\mathbf{s}(j) \in S$	Performance	Output $\mathbf{y}(j) \in Y$
0	$\mathbf{x}(0)=(\mathbf{u}(0), \mathbf{w}(0))$	$\mathbf{s}(0)$ given		$\mathbf{y}(0)$

1	$x(1)=(u(1),w(1))$	$s(1)=F(s(0), x(o))$	$PI(1)\epsilon s(1)$	$y(1)=G(s(1))$
...	...	...	...	...
t	$x(t)=(u(t),w(t))$	$s(t)=F(s(t-1), x(t-1))$	$PI(t)\epsilon s(t)$	$y(t)=G(s(t))$ $FM(t)$

Table 1. Behavior of a system under experiment  $e(t) = (x(t), s(0), t)$ .

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### **Biographical Sketch**

**Lucien Duckstein** was a professor of Systems and Industrial Engineering and also of Hydrology and Water Resources at the University of Arizona Tucson, USA, from 1962 to 1997.

He has then become a professor emeritus at the same institution and has since returned to his native city, Paris, France, as a professor at ENGREF (French Institute of Agronomy, Water Resources and Forestry). His research areas cover multiobjective analysis, decision theory, statistical and Bayesian decision theory, fuzzy logic with applications to hydrology and water resources.