SYMBOLIC THERMOECONOMIC ANALYSIS OF ENERGY SYSTEMS

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Summary

Symbolic thermoeconomics is a methodology for the analysis of the productive structure, and the natural resources consumption process in energy systems. It appears as a technique, based on the Exergy Cost Theory, to obtain general equations, which relate the overall efficiency of an energy system and other thermoeconomic variables as fuel, product, exergy cost, with the efficiency of each component which forms it. By mean of the equations obtained, it is possible to analyze the influence of the individual consumption of each component on the total amount of external resources required to obtain a product. Therefore, the productive structure and the cost formation process of the products are explained.

1. Introduction

The cost accounting methodologies, as Exergy Cost Theory, propose methods to determine the amount of resources required for obtaining a product. They are based on cost assessment rules, which attribute to the useful product the resource cost of each component, and distribute its costs proportionally to its exergies. They are mainly numerical techniques that calculate the cost values in an accurate way, by solving sets of linear equations, but they cannot identify the causes of the cost formation process.

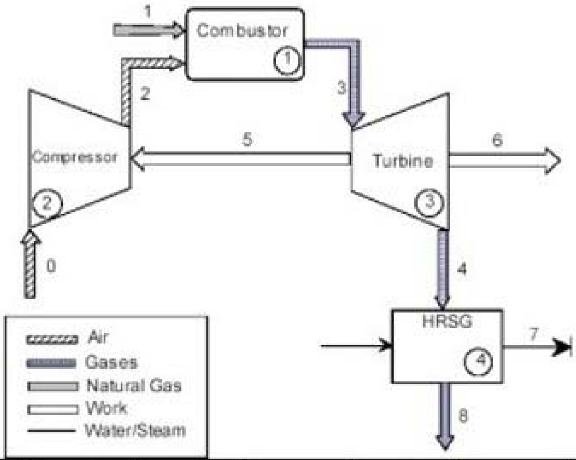
Suppose the thermal system shown in Figure 1. In this article it is explained the procedure for obtaining the global efficiency of the system as a function of the efficiencies, ζ_i, of its components is:

$$\zeta_T = \frac{\zeta_1\left(y_2\left(1-y_3\right)\zeta_2+\left(1-y_3\right)\zeta_3\right)}{1-y_2 y_3 \zeta_2 \zeta_3}$$

where terms y_i represents the bifurcation ratios.

This poses some initial questions: Could we obtain such a formula in a general way? What conditions must fulfill our method to assess it? If such a formula exists, questions of the type, *what happens if?*, could be simplified. Thus we could compute, for example, how a variation in the efficiency of a component modify the efficiency of the whole plant. With general formulae we can get general solutions to general problems. The proposed analysis provides a set of valuable tools for the cost accounting, diagnosis, optimization and synthesis of energy systems.

Symbolic computation packages, like *Mathematica* or *MatLab*, could be used to solve a wide variety of technical computing problems, such as to obtain both analytical and numerical solutions of linear systems of equations. If we bring together Exergy Cost Theory and Symbolic Computation then it will be possible to find out a general way to obtain formulae like that shown above. This was the reason to call this methodology *Symbolic Thermoeconomics*. Some of its applications are presented in the bibliography listed at the end of this article.



Flow (kW)	1	2	3	4	5	6	7	8
Design	11781	2704	9614	3831	2977	2500	2355	388

Figure 1. Gas turbine cycle diagram.

In order to follow the arguments of the proposed methodology, we use a simple example of a power plant, whose block diagram, design and operational exergy flow values are shown in Figure 1. The plant is made up of a cogeneration gas turbine cycle, and uses the turbine outlet gases as thermal energy in a heat recovery steam generator, that produces steam (flow #7) together with the electric energy produced in the turbogenerator (flow #6).

2. The Fuel-Product Model

The first stage to identify the cost process formation consists of building, from the physical structure of the plant, a productive scheme which shows where the product of each component is used and the origin of the resources of each component.

The problem of the productive structure identification is closely related to *Leontief's input-output economic analysis*. It consists of a qualitative and quantitative analysis of the relations that link the flows of goods and services between the components of an economic unit, in order to study its structural characteristics.

An equivalent model could be applied to thermal systems. It can be represented by a fuel/product diagram, such as shown in Figure 2 and Table 1, for the plant of the example. It is also called in other thermoeconomic methodologies Functional Diagram.

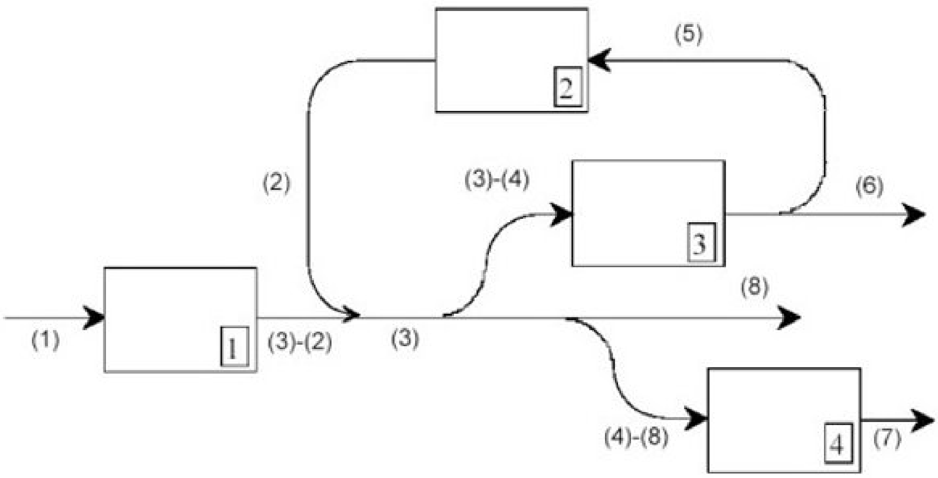


Figure 2. Fuel/product diagram.

	F_0	F_1	F_2	F_3	F_4	**Total**
P_0	0	E_1	0	0	0	E_1
P_1	0	0	0	$r_1(E_3-E_4)$	$r_1(E_4-E_8)$	$r_1(E_3-E_8)$
P_2	0	0	0	$r_2(E_3-E_4)$	$r_2(E_4-E_8)$	$r_2(E_3-E_8)$
P_3	E_6	0	B_5	0	0	E_5+E_6
P_4	E_7	0	0	0	0	E_7
Total	E_6+E_7	E_1	E_5	E_3-E_4	E_4-E_8	

Table 1. Fuel/product table.

Note that, not all products of components #1 and #2 are used as resources in components #3, #4 or final product, the flow #8 is a residue generated in components #1 and #2.

$$r_1 = \frac{E_3 - E_2}{E_3} \qquad r_2 = \frac{E_2}{E_3} \qquad r_1 + r_2 = 1$$

In accordance with this model the production of one component is used as fuel of another component or as a part of the total production of the plant:

$$P_i = E_{i0} + \sum_{j=1}^{n} E_{ij}\text{ , i = 0,1,…,n} \qquad (1)$$

where E_{ij} is the production portion of the *i*-th component that fuels the *j*-th component. In the above expression, we consider the component 0, as the system environment, then E_{i0} represents the production portion of the component *i* which leads to the final product, coming from the environment to the component *i*.

On the other hand, the resources entering each component, could be expressed as:

$$F_i = E_{0\mathrm{i}} + \sum_{j=1}^{n} E_{ji}\text{ , i = 0,1,…,n} \qquad (2)$$

where $E_{0\mathrm{i}}$ represents the external resources entering to the plant, which go into the *i*-th component.

Therefore, the total fuel and product of the system could be expressed as:

$$F_T \equiv P_0 = \sum_{j=1}^{n} E_{0j} \qquad P_T \equiv F_0 = \sum_{j=1}^{n} E_{j0} \qquad (3)$$

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Biographical Sketch

César Torres Cuadra is research contributor at the CIRCE foundation, Center of Research for Energy Resources and Consumption, Zaragoza, Spain. C. Torres received the Bachelor Diploma in Mathematics from the University of Zaragoza in 1984. He worked as researcher in the ITA—Technological Institute of Aragon – and he performed graduate studies in Mechanical Engineering with major in Energy Optimization in the University of Zaragoza, lading to the Ph.D. degree in 1991. He works in ENDESA, one of the main Spanish utilities, at the Telecommunications and Control System Division, as software engineer on electric network and generation control systems projects. His research activity is related to the development of thermoeconomic analysis of energy systems methodologies. He has an extensive number of publications in journals and international conferences.