FLUID MECHANICS AND MULTIPHASE FLOW IN PIPELINES

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Contents

Summary

A large number of life support systems involve fluid flow in channels. This chapter is aimed at a basic understanding of fluid mechanics and multiphase flow in pipes and channels. This is reached by formulating the classical balance equations of fluid mechanics, namely mass, momentum, energy and entropy, and applying them to channel flow. Usually this approach is intended to, under simplifying assumptions, provide a system of differential equations describing the evolution of the main flow properties (e.g. velocity, pressure, temperature, concentration, etc.) along the channel length and/or time. Several difficulties arise in such an endeavor, even when the fluid consists of a chemically homogeneous substance in single-phase flow. First, *constitutive laws* are required to specify the thermo-dynamical and thermo-physical behavior of the fluid, such as provided by equations of state, stress-strain relationships, drag law, heat propagation law, etc.

These laws make it possible to close the resulting mathematical problem in terms of an equal number of equations and unknowns. Disregarding the axiomatic nature of the balance equations itself, the specification of constitutive relations is based directly on previous accumulated experimental evidence on single-phase flows, e.g. drag laws in turbulent flow.

That is why fluid mechanics is much more dependent upon a fruitful relation between theory and experiment than other branches of mechanics. Second, appropriate boundary conditions are required. If the fluid consists of two or more phases that are to be treated separately, conditions to be satisfied at each interface and wall at any time instant must be specified. This chapter presents the balance equations governing single and multiphase channel flows under the unifying approach of continuum mechanics. Appropriate boundary conditions at interfaces are developed as part of the derivation. Some important constitutive laws are presented for the sake of completeness.

1. Introduction

Fluids, broadly understood as liquids and gases, are defined as substances that do not resist to an external shear (i.e. they flow). However, in many practical situations the flow of a fluid is not ultimately caused by the application of shear itself but by pressure, gravity, buoyancy or inertia instead.

An example of shear-driven or *Couette* flows occurs in lubricating systems such as bearings. Blood flow inside an artery and water flow in the feeding pipe of your house are examples of pressure-driven flows or *Poiseuille* flows. Downhill river flow is of course driven by gravity but ascending air flow in the atmosphere may be caused by buoyancy (warm air lighter than surrounding air). The flow inside the impeller of a centrifugal pump is caused by the inertia provided by the impeller.

Life support systems, either natural or human engineered, comprise a quite large number of devices or situations where fluid flow is involved. They may act as an energy supplier as in a boiler or a turbine, cooling agent, pressure transmitter, material transporters, etc. Fluids are themselves life support agents and may carry live beings on them. Hence, they are present in nearly all life systems and play a central role in mechanical, chemical and process engineering.

An essential classification of channel flows regards the number of phases present. *Phase* is understood as the aggregation state of matter and is usually referred to as solid, liquid or gas. Thus, one may have a single-phase liquid or gas flows, a two-phase gas-liquid or liquid-liquid flow, a three phase gas-liquid-solid or liquid-liquid-gas flow, and so on. This classification has of course nothing to do with the number of chemical species present.

In multiphase flow, the phases are separated from each other by an *interface*, where the thermo-dynamical and thermo-physical properties change abruptly. Multiphase flow may occur with or without phase-change. Typical multiphase flows with phase change occur in boilers and condensers; examples of flows without phase change are provided by hydraulic or pneumatic transportation systems.

This chapter is aimed at a basic understanding of the equations governing fluid motion. For this purpose we assume that it behaves as a continuum distribution of matter and therefore adopt the approach of *continuum mechanics*. The governing equations to be presented are the mass, momentum, energy and entropy balances, first in integral then in differential form. The presentation follows the classical view of mechanics, according to which any relativistic or nuclear effects in fluid motion are negligible.

In single-phase channel flows, the presence of discontinuities at the walls is noticeable. In multiphase flows, phase discontinuities or interfaces lie also inside the flow field. The continuum mechanics approach adopted here deals with such occurrences by assuming that the interfaces are *discontinuity surfaces*. For simplicity, the equations governing single-phase flows are considered first. Then the approach is extended to two-phase and multiphase flows, where a rigorous derivation of the equations to be satisfied at interfaces (i.e. the *jump conditions*) is presented.

2. Mathematical Tools

2.1. Displacement Speed of a Surface

Let $\mathcal{V}$(t) be any mobile single-phase control volume limited by a closed surface $\mathcal{A}$(t), Figure 1.

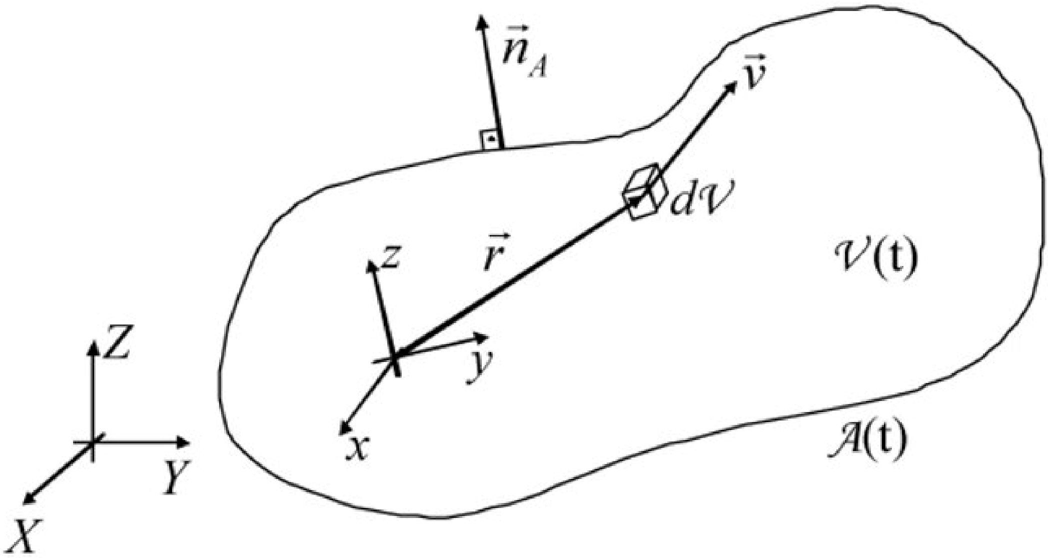

Figure 1. Mobile single-phase control volume, its attached xyz frame and inertial *XYZ* frame

Let $\mathcal{A}$(t) be implicitly defined by the relation:

$$f_A(x, y, z, t) = 0 \qquad (1)$$

where *(x, y, z)* are the coordinates of a generic point in the attached *xyz* coordinate system, which may be inertial or non-inertial. Let $\vec{n}_A$ be the unit vector normal to $\mathcal{A}$(t) pointing to the outside of $\mathcal{V}$(t); $\vec{n}_A$ is related to $f_A(x, y, z, t)$ by:

$$\vec{n}_A = \frac{\text{grad } f_A}{|\text{grad } f_A|} \qquad (2)$$

The velocity of an arbitrary point $A(x_A, y_A, z_A)$ of $\mathcal{A}$(t) as measured in *xyz* frame is given by:

$$\vec{v}_A = \frac{dx_A}{dt}\mathbf{i} + \frac{dy_A}{dt}\mathbf{j} + \frac{dz_A}{dt}\mathbf{k} \qquad (3)$$

The time derivative of Eq. (1) is:

$$\frac{df_A}{dt} = \frac{\partial f_A}{\partial t} + \vec{v}_A \cdot \text{grad } f_A = 0 \quad (4)$$

and from Eqs. (2) and (4) one can conclude:

$$\vec{v}_A \cdot \vec{n}_A = -\frac{1}{|\text{grad } f_A|}\frac{\partial f_A}{\partial t} \quad (5)$$

which is a scalar quantity known as the *displacement speed* of $\mathcal{A}$(t). Note that this speed depends only on the relation describing the *shape* of the surface, whereas $\vec{v}_A$ also depends on the existence of a tangential motion of point A along the surface, i.e.:

$$\vec{v}_A = (\vec{v}_A \cdot \vec{n}_A)\vec{n}_A + \vec{v}_A^{\,t} \quad (6)$$

where $\vec{v}_A^{\,t}$ is the vector component of $\vec{v}_A$ tangent to $\mathcal{A}$(t). This result can be extended to a fixed two-phase control volume $\mathcal{V}_1$(t) + $\mathcal{V}_2$(t), such as shown in Figure 2.

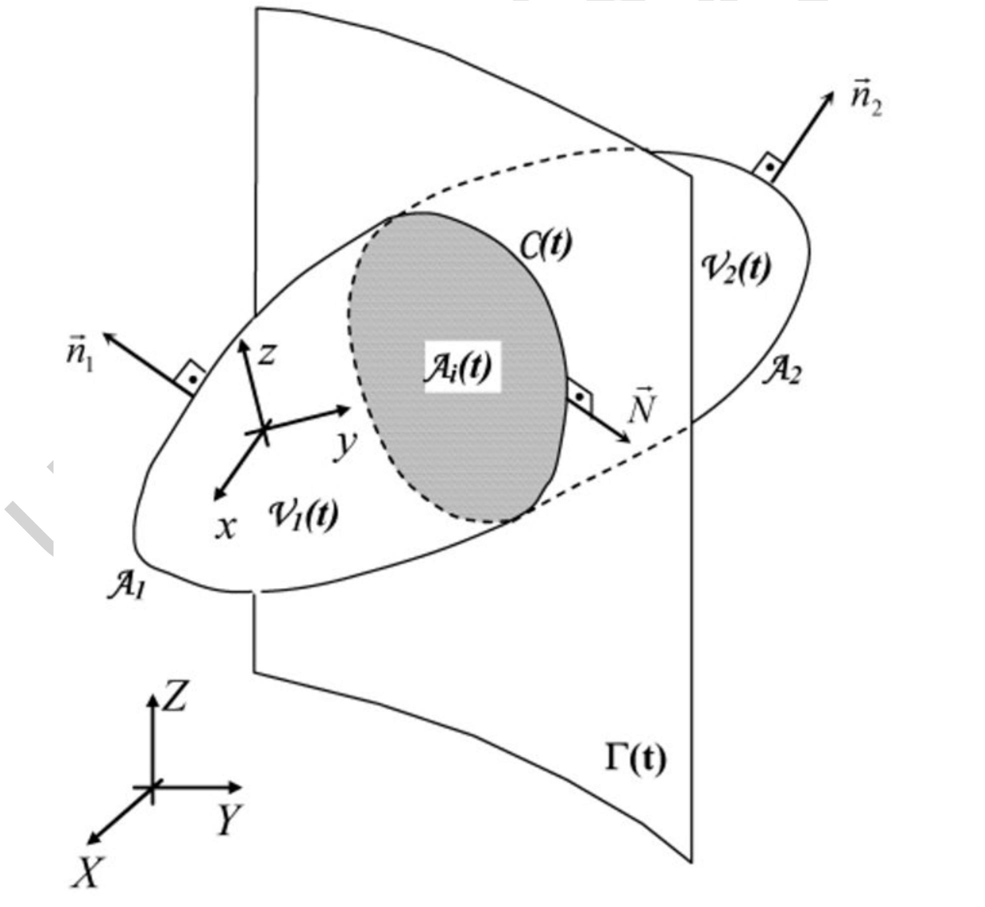

Figure 2. A fixed two-phase control volume with mobile interface

The phase volumes $\mathcal{V}_1$(t) and $\mathcal{V}_2$(t) are not individually fixed once they are separated by a mobile interfacial area $\mathcal{A}_i$(t). However, the sum $\mathcal{V}_1$(t) + $\mathcal{V}_2$(t) is a fixed volume.

Thus, the displacement speeds of $\mathcal{A}_1$ and $\mathcal{A}_2$ are zero but the displacement speed of $\mathcal{A}_i(t)$ is not. As $\mathcal{A}_i(t)$ moves, so does the closed curve $\mathcal{C}(t)$. The velocity of arbitrary points of $\mathcal{A}_i(t)$ and $\mathcal{C}(t)$ are:

$$\vec{v}_{A_i} = \left(\vec{v}_{A_i} \cdot \vec{n}_i\right)\vec{n}_i + \vec{v}^{\,t}_{A_i} \tag{7}$$

$$\vec{v}_C = \left(\vec{v}_{A_i} \cdot \vec{n}_i\right)\vec{n}_i + \left(\vec{v}_C \cdot \vec{N}\right)\vec{N} \tag{8}$$

where $\vec{v}^{\,t}_{A_i}$ is the tangential vector component of $\vec{v}_{A_i}$, $\vec{n}_i$ is the unit vector normal to $\mathcal{A}_i(t)$ (see Figure. 3), $\vec{N}$ the surface unit vector normal to $\mathcal{C}(t)$, $\vec{v}_{A_i} \cdot \vec{n}_i$ is the displacement speed of $\mathcal{A}_i(t)$ and $\vec{v}_C \cdot \vec{N}$ is the displacement speed of curve $\mathcal{C}(t)$ on the tangent surface $\Gamma(t)$; $\vec{v}_C \cdot \vec{N}$ is related to $\vec{v}_{A_i} \cdot \vec{n}_i$ by:

$$\vec{v}_C \cdot \vec{N} = -\left(\vec{v}_{A_i} \cdot \vec{n}_i\right)\frac{\vec{n}_i \cdot \vec{n}_k}{\vec{N} \cdot \vec{n}_k} \qquad k = 1,2 \tag{9}$$

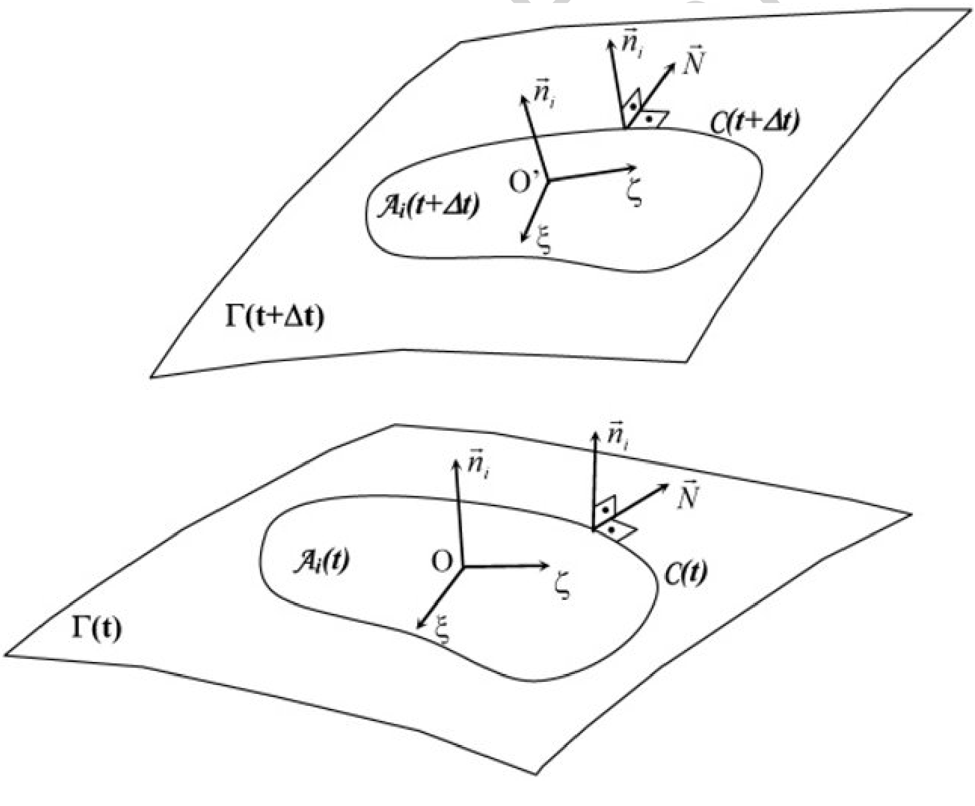

Figure 3. Surface coordinates unit vectors and displacement of the interface

2.2. Leibniz Rule

$$\frac{d}{dt}\int_{\mathcal{V}(t)}\varphi(x,y,z,t)\rho(x,y,z,t)d\mathcal{V} = \int_{\mathcal{V}(t)}\frac{\partial(\rho\varphi)}{\partial t}d\mathcal{V} + \oint_{\mathcal{A}(t)}\varphi\rho(\vec{v}_A\cdot\vec{n}_A)d\mathcal{A} \qquad (10)$$

where $\varphi(x,y,z,t)$ is any scalar or vector field such as a fluid property per unit mass, $\rho(x,y,z,t)$ the specific mass of the fluid and $(\vec{v}_A\cdot\vec{n}_A)$ is the displacement speed of $\mathcal{A}$(t). The symbol $\oint_{\mathcal{A}(t)}$ is used throughout this chapter to indicate the integral over a *closed* surface $\mathcal{A}$(t).

2.3. Gauss-Green Theorems

If $\vec{B}$ is a space vector and $\mathbf{T}$ a tensor then, for the control volume shown in Figure. 1:

$$\oint_{\mathcal{A}(t)}\vec{B}\cdot\vec{n}_A d\mathcal{A} = \int_{\mathcal{V}(t)}\mathrm{div}\,\vec{B}\,d\mathcal{V} \qquad (11a)$$

$$\oint_{\mathcal{A}(t)}\vec{n}_A\cdot\mathbf{T}\,d\mathcal{A} = \int_{\mathcal{V}(t)}\mathrm{div}\,\mathbf{T}\,d\mathcal{V} \qquad (11b)$$

For two-phase control volumes such as illustrated in Figures. 2 and 3, we further need a few surface theorems. If ϑ is a scalar then (Figure. 3):

$$\oint_{C(t)}\vartheta\,\vec{N}\,dC = \int_{\mathcal{A}_i(t)}\left[\mathrm{grad}_s\vartheta-(\mathrm{div}_s\vec{n}_i)\vartheta\,\vec{n}_i\right]d\mathcal{A} \qquad (12a)$$

$$\oint_{C(t)}\vec{r}\times\vartheta\,\vec{N}\,dC = \int_{\mathcal{A}_i(t)}\vec{r}\times\left[\mathrm{grad}_s\,\vartheta-(\mathrm{div}_s\,\vec{n}_i)\vartheta\,\vec{n}_i\right]d\mathcal{A} \qquad (12b)$$

$$\oint_{C(t)}\vec{B}\cdot\vec{N}\,dC = \int_{\mathcal{A}_i(t)}\mathrm{div}_s\vec{B}^t\,d\mathcal{A} \qquad (12c)$$

where:

$$\mathrm{grad}_s\,\vartheta = \mathbf{i}_\xi\frac{\partial\vartheta}{\partial\xi}+\mathbf{i}_\zeta\frac{\partial\vartheta}{\partial\zeta}\quad \textit{surface gradient} \qquad (13a)$$

$$\mathrm{div}_s\,\vec{B}^t = \frac{\partial B_\xi}{\partial\xi}+\frac{\partial B_\zeta}{\partial\zeta}\qquad \textit{surface divergent} \qquad (13b)$$

$$\mathrm{div}_s \vec{n}_i = \frac{1}{R_\xi} + \frac{1}{R_\zeta} = \frac{2}{R} \qquad \textit{local curvature of } \mathcal{A}_i(t) \tag{13c}$$

2.4. Transport Rates

2.4.1. Rate of Flow

The most distinctive feature of fluid mechanics in comparison with other branches of mechanics is the concept of flow rate. The (volume) flow rate Q is the volume per unit time of fluid passing *through* an (imaginary) curved surface $\mathcal{A}$(t). Similarly, the mass flow rate is the mass per unit time of fluid crossing $\mathcal{A}$(t). If $\mathcal{A}$(t) has a unit vector $\vec{n}_A$ normal to each surface element $d\mathcal{A}$, then Q will depend only upon the component $(\vec{v} - \vec{v}_A)\cdot\vec{n}_A$ of the *relative* fluid velocity across $d\mathcal{A}$ i.e.

$$Q = \int_{\mathcal{A}(t)} (\vec{v} - \vec{v}_A)\cdot\vec{n}_A d\mathcal{A} \quad = \quad \textit{volume flow rate} \tag{14}$$

$$\int_{\mathcal{A}(t)} \rho(\vec{v} - \vec{v}_A)\cdot\vec{n}_A d\mathcal{A} \quad = \quad \textit{mass flow rate} \tag{15}$$

where $\vec{v}$ is the fluid velocity measured in the xyz reference frame.

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Biographical Sketch

Antonio Carlos Bannwart, born in 1955, is Professor of Fluid Mechanics and Multiphase Flow at the Department of Petroleum Engineering of the State University of Campinas-UNICAMP, Brazil, since 1978. He obtained the *Doctorat* at the Institut Nationale Polytechnique de Grenoble (INP-Grenoble, France), in 1988. In 1996 he was a Visiting Researcher at the University of Minnesota, USA. His major research activity is focused on multiphase flow applied to petroleum exploration and production. He contributes regularly with the major journals and conferences of his area. Dr. Bannwart has 30 years experience teaching fluid mechanics to undergraduate mechanical engineering students and 20 years teaching multiphase flow to graduate petroleum engineering students at UNICAMP.