CLOSED-LOOP BEHAVIOR OF CONTINUOUS LINEAR TIME-INVARIANT SYSTEMS

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Keywords: Process or plant, Sensor, Actuator, Controller, Feedback path, Reference signal, Manipulating or control signal, Control error, Disturbance rejection, Tracking control, Sensitivity function, Characteristic polynomial, Stability, Performance index, Steady-state error, Three-term controller or PID controller, Gain factor, Integral time constant, Derivative time constant.

Contents

Summary

This article describes the static and dynamic behavior of linear time-invariant continuous single input/single output systems in the basic closed-loop control structure. Aspects of tracking control, disturbance rejection control and reduction of sensitivity to parameter variations in the system are discussed. How to avoid steady-state errors in the closed-loop system by selecting appropriate structures of the open-loop system or the controller is explained. Standard controller types and their characteristics are presented.

1. Dynamic Behavior of the Closed-Loop Control System

The basic difference between open- and closed-loop control systems has already been dealt with in the introductory contributions (see *Elements of Control Systems* and *Introduction to Basic Elements*). Figure 1 shows the generic block diagram of a closed-loop control system with its four essential elements: Process or plant, sensor or measuring element, controller and actuator. Often it is recommended to combine the plant and sensor together into one block as well as considering the controller and actuator as being one unit. This leads to the simplified block diagram structure of Figure 2.

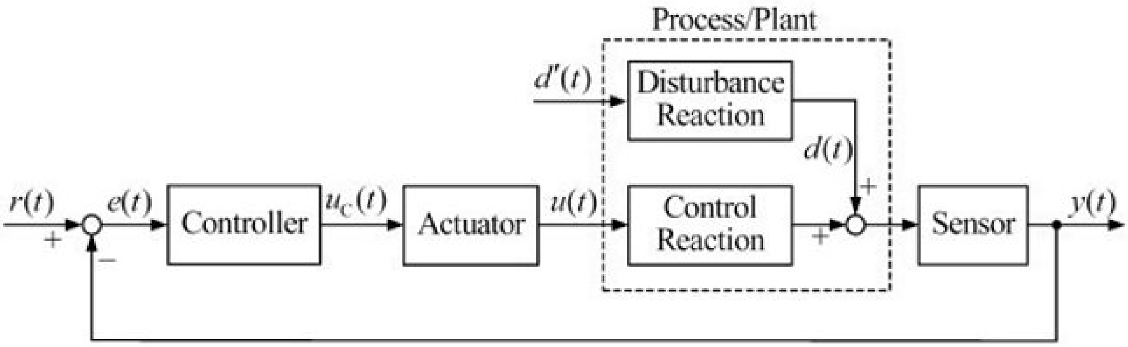

Figure 1. Generic block diagram of a closed-loop control system (Signals: *r* reference/command signal or set point; *u* control signal or manipulating variable; *y* controlled variable or output signal; *d* disturbance; *e* error signal or control error signal; $u_C(t)$ controller output signal)

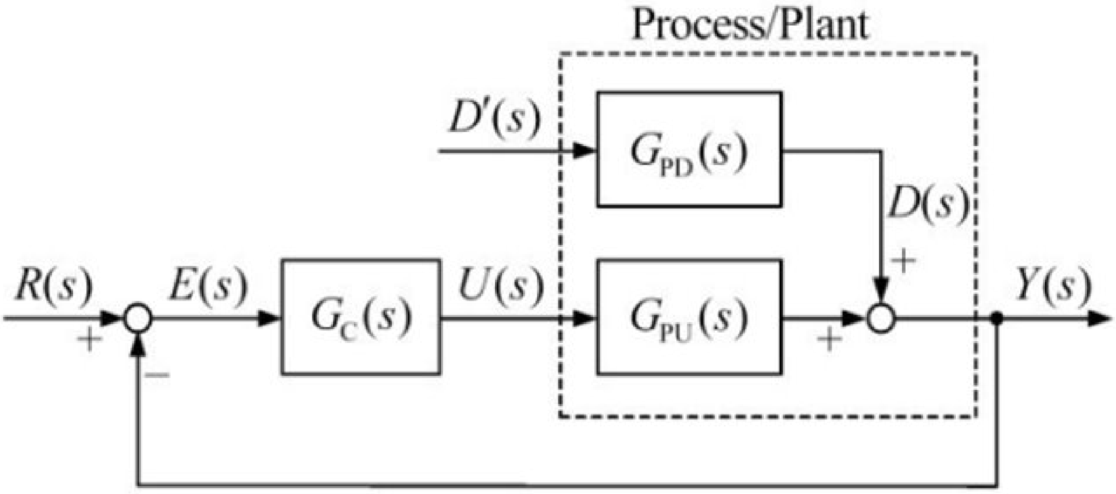

Figure 2. Simplified block diagram of a closed-loop control system and corresponding transfer functions of the controller or control unit $G_C(s)$ and the plant $G_{PU}(s)$ and $G_{PD}(s)$, respectively

At the first summing junction the output signal or controlled variable $y(t)$ which arrives via the feedback path is subtracted from the reference signal $r(t)$. The result is denoted as control error $e(t)$, or simply as error, which is further processed in the controller unit described by the transfer function $G_C(s)$. The controller unit can be realized either by an analog circuit or by a digital computer. The output of the controller unit is called the manipulating or control signal $u(t)$ that acts on the plant. The influence of $u(t)$ on the plant output $y(t)$ in the case of an undisturbed plant is characterized by the actuating transfer function $G_{PU}(s)$, whereas in the disturbed case all disturbances $d_i'(t)$ $(i = 1, 2, \ldots)$ occurring at the plant can be added via elements representing the disturbance transfer functions $G_{PD_i}(s)$ at the second summing junction to the plant output. Figure 2 shows the case of one disturbance, where $G_{PD}(s)$ is the disturbance transfer function of the plant. The dynamic behavior of the plant thus depends on its response to the manipulating signal $u(t)$ and the disturbance signal $d'(t)$, described by $G_{\mathrm{PU}}(s)$ and $G_{\mathrm{PD}}(s)$.

The closed-loop system drives the plant via the manipulating signal if there is a nonzero control error $e(t)$ at the first summing junction due to either a disturbance $d'(t)$ or changes of the reference signal $r(t)$. In case the steady-state $e(t)=0$ is reached, the plant is no longer driven, since its response $y(t)$ is already identical with the desired response $r(t)$. Compared to open-loop systems, closed-loop systems have the obvious advantage that they are less sensitive to disturbances and are able to attenuate or reject these disturbances. Furthermore, they are able to follow a desired reference signal.

The controlled variable of a closed loop, or feedback control, system according to Figure 2 is given by the frequency-domain description

$$Y(s)=\frac{G_{\mathrm{PD}}(s)}{1+G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)}\,D'(s)+\frac{G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)}{1+G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)}\,R(s). \tag{1}$$

Using this equation, both tasks of a feedback control system can be defined:

a) For $R(s)=0$ the case of *disturbance rejection control* follows from Eq. (1) by the closed-loop transfer function

$$G_{\mathrm{D}}(s)=\frac{Y(s)}{D(s)}=\frac{G_{\mathrm{PD}}(s)}{1+G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)}. \tag{2}$$

This case can be generalized as being the task of keeping the reference signal at a constant value, $r(t)=\text{const.}$, despite disturbances acting onto the plant.

b) For $D'(s)=0$ the case of *tracking control* follows from Eq. (1) by the closed-loop transfer function

$$G_{\mathrm{R}}(s)=\frac{Y(s)}{R(s)}=\frac{G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)}{1+G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)}. \tag{3}$$

Both closed-loop transfer functions, Eqs. (2) and (3), contain the same common factor, often denoted as dynamical control factor

$$G^{*}(s)=\frac{1}{1+G_{0}(s)}, \tag{4}$$

where

$$G_{0}(s)=G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s) \tag{5}$$

characterizes, by omitting the negative sign, the behavior of the open-loop system, when cutting any signal path of the closed loop.

2. Sensitivity of Feedback Control Systems to Parameter Variations

Besides providing good tracking behavior for a variable or constant reference signal $r(t)$ the primary purpose of a feedback control system is to minimize the effect of disturbances $d_i'(t)$ and to reduce the sensitivity to parameter variations in the plant transfer function $G_{\mathrm{PU}}(s)$. Every plant is subject to parameter changes or uncertainties during its operation, for example due to changes of the environmental conditions, imprecise knowledge of process parameters or ageing. Unlike the open-loop system, the closed-loop system is able to sense the changes in the output signal due to the process changes and attempts to correct the output.

To illustrate the effect of parameter variations, consider the case when the plant transfer function $G_{\mathrm{PU}}(s)$ is changed to $G_{\mathrm{PU}}(s)+\Delta G_{\mathrm{PU}}(s)$. Then, Eq. (3) changes to

$$Y(s)+\Delta Y(s)=\frac{G_{\mathrm{C}}(s)[G_{\mathrm{PU}}(s)+\Delta G_{\mathrm{PU}}(s)]}{1+G_{\mathrm{C}}(s)[G_{\mathrm{PU}}(s)+\Delta G_{\mathrm{PU}}(s)]}\,R(s), \tag{6}$$

and the change in the output signal is

$$\Delta Y(s)=\frac{G_{\mathrm{C}}(s)\Delta G_{\mathrm{PU}}(s)}{([1+G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)+G_{\mathrm{C}}(s)\Delta G_{\mathrm{PU}}(s)][1+G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s)])}\,R(s)\,. \tag{7}$$

Since usually $G_{\mathrm{C}}(s)G_{\mathrm{PU}}(s) >> G_{\mathrm{C}}(s)\Delta G_{\mathrm{PU}}(s)$, Eq. (7) using Eq. (5) reduces to

$$\Delta Y(s)=\frac{G_{\mathrm{C}}(s)\Delta G_{\mathrm{PU}}(s)}{[1+G_0(s)]^2}\,R(s)\,. \tag{8}$$

Thus we can conclude that the change in the output signal of the feedback control system is reduced by the factor $1+G_0(s)=G^{*-1}(s)$, where the magnitude of $G^{*-1}(s)$ is usually much greater than one in the range of frequencies of interest.

The dependence of the closed-loop system's characteristic on those of a particular element of the loop can be expressed by the corresponding *sensitivity function*. For the above considered case of changes $\Delta G_{\mathrm{PU}}(s)$ in the plant transfer function, the sensitivity of the closed-loop transfer function $G_{\mathrm{R}}(s)$ is defined as

$$S_{G_{\mathrm{PU}}}^{G_{\mathrm{R}}}(s) = \frac{\Delta G_{\mathrm{R}}(s)/G_{\mathrm{R}}(s)}{\Delta G_{\mathrm{PU}}(s)/G_{\mathrm{PU}}(s)} = \frac{\partial G_{\mathrm{R}}/G_{\mathrm{R}}}{\partial G_{\mathrm{PU}}/G_{\mathrm{PU}}} = \frac{\partial \ln G_{\mathrm{R}}}{\partial \ln G_{\mathrm{PU}}} \tag{9}$$

Using Eq. (3), the sensitivity according to Eq. (9) is

$$S_{G_{\mathrm{PU}}}^{G_{\mathrm{R}}}(s) = \frac{\partial G_{\mathrm{R}}}{\partial G_{\mathrm{PU}}} \frac{G_{\mathrm{PU}}}{G_{\mathrm{R}}} = \frac{G_{\mathrm{C}}}{(1+G_{\mathrm{C}}G_{\mathrm{PU}})^2}$$
$$\frac{G_{\mathrm{PU}}}{G_{\mathrm{C}}G_{\mathrm{PU}}/(1+G_{\mathrm{C}}G_{\mathrm{PU}})} = \frac{1}{1+G_{\mathrm{C}}G_{\mathrm{PU}}} \equiv G^* \tag{10}$$

The result obtained in Eq. (10) shows that from a sensitivity viewpoint it appears desirable to design the magnitude of the open-loop transfer function $G_0(s) = G_{\mathrm{C}}(s)\, G_{\mathrm{PU}}$ as large as possible. However, this will only be possible over a certain frequency range. Thus it is important to recognize that sensitivity or insensitivity of a feedback control system can only be obtained over certain frequency bands.

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Biographical Sketch

Heinz Unbehauen is Professor Emeritus at the Faculty of Electrical Engineering and Information Sciences at Ruhr-University, Bochum, Germany. He received the Dipl.-Ing. degree from the University of Stuttgart, Germany, in 1961 and the Dr.-Ing. and Dr.-Ing. habil. degrees in Automatic Control from the same university in 1964 and 1969, respectively. In 1969, he was awarded the title of Docent and in 1972 he was appointed as Professor of control engineering in the Department of Energy Systems at the University of Stuttgart. Since 1975, he has been Professor at Ruhr-University of Bochum, Faculty of Electrical Engineering, where he was head of the Control Engineering Laboratory until February 2001. He was Dean of his faculty in 1978/79. He was a Visiting Professor in Japan, India, China and the USA. He has authored and co-authored over 400 journal articles, conference papers and 7 books. He has delivered many invited lectures and special courses at universities and companies around the world. His main research interests are in the fields of system identification, adaptive control, robust control and process control of multivariable systems. He is Honorary Editor of IEE Proceedings on Control Theory and Application and System Science, Associate Editor of Automatica and serves on the Editorial Board of the International Journal of Adaptive Control and Signal Processing, Optimal Control Applications and Methods (OCAM) and Systems Science. He also served as associate editor of IEEE-Transactions on Circuits and Systems as well as Control-Theory and Advanced Technology (C-TAT). He is also an Honorary Professor of Tongji University Shanghai. He has been a consultant for many companies, as well as for public organisations, e.g., UNIDO and UNESCO. He is a member of several national and international professional organisations and a Fellow of IEEE.