EXTENDED CONTROL STRUCTURES

Boris Lohmann
Institut für Automatisierungstechnik, Universität Bremen, Germany

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Contents

Summary

The combination of state feedback with a state observer provides a powerful tool for designing linear control systems. However, the steady state behavior in the presence of model uncertainty or permanent disturbances turns out to be unsatisfactory.

This disadvantage can be overcome with the help of an *overlying PI-controller*. Another structural extension of the control systems introduced so far is the introduction of a *dynamic model-based pre-compensator*. It allows the designer to separately shape the disturbance- and the input-output behavior of the control system.

1. Steady State Behavior under realistic assumptions

Figure 1 shows the arrangement of a control plant with state observer and state feedback, as introduced in the previous articles. The pre-compensator gain g is selected so that the control output y converges towards the (constant) reference r as $t \to \infty$.

The only disturbance acting on the system is the unknown *initial state* $\mathbf{x}(t_0) = \mathbf{x}_0$. As the initial time t_0 of the consideration can be set arbitrarily, we can establish that this control system will eliminate those disturbances vanishing from a certain time t_0 on.

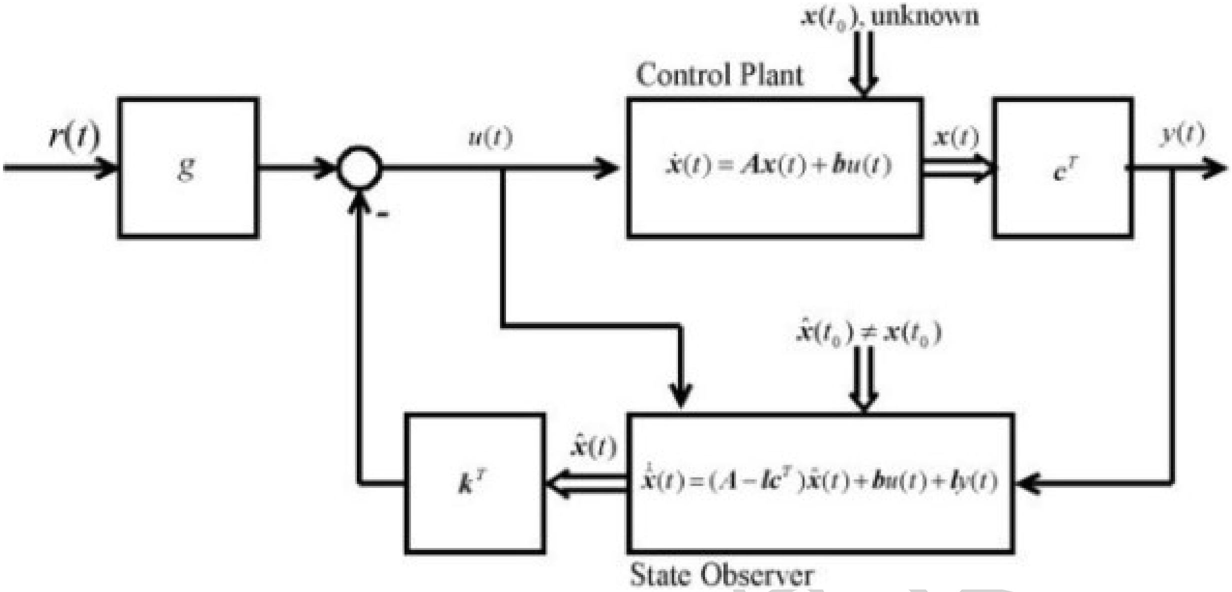

Figure 1: Close-loop system with state observer and state feedback

1.1 External Disturbances

The situation changes in case of *permanent disturbances*. Then, in many cases the influence of a disturbance variable $z(t)$ on the state differential equation can be modeled by a linear term, and the state equations can be extended by a corresponding summand $\mathbf{e}z(t)$,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t) + \mathbf{e}z(t)\,, \qquad (1)$$

$$y = \mathbf{c}^{\mathrm{T}}\mathbf{x}\,. \qquad (2)$$

The input-output behavior from the disturbance input z to the control output y can then be expressed (after Laplace transform):

$$Y(s) = \mathbf{c}^{\mathrm{T}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}U(s) + \mathbf{c}^{\mathrm{T}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{e}Z(s)\,. \qquad (3)$$

If a pure state feedback controller without observer is used for the control of this plant, the input-output relation of the closed-loop system becomes

$$Y(s) = \underbrace{\mathbf{c}^{\mathrm{T}}(s\mathbf{I} - \mathbf{A} + \mathbf{b}\mathbf{k}^{\mathrm{T}})^{-1}\mathbf{b}g}_{G_r(s)}\,R(s) + \underbrace{\mathbf{c}^{\mathrm{T}}(s\mathbf{I} - \mathbf{A} + \mathbf{b}\mathbf{k}^{T})^{-1}\mathbf{e}}_{G_z(s)}\,Z(s)\,. \qquad (4)$$

The second summand describes the effect of the disturbance on the control output. In general, the *disturbance transfer function* $G_z(s)$ will not equal zero and even simple constant disturbances $z(t) = \mathrm{const.}$ will cause permanent steady state error, i.e. a permanent nonzero difference between y and r. Since all poles of $G_z(s)$ obviously are control eigenvalues, we can at least state that the disturbance z does not affect the general stability properties of the closed-loop system.

Finite disturbances will cause finite reactions in the control output *y*, provided the amplitude of the disturbance does not cause system variables to reach their natural boundaries or the linear model to loose validity for other reasons. These observations are also true when including an observer. In case of *several* disturbances $z_1(t),...,z_m(t)$, they can be modeled by several corresponding summands $\mathbf{e}_\nu z_\nu(t)$ in the state equations.

In case of finite permanent disturbances z acting according to (1) on the plant within a closed-loop system (with or without observer), the control output y will in general no longer follow the reference signal r, i.e. steady state error will occur. Stability is not affected.

The problem is not so easy surveyed if the influence of disturbances is in a *nonlinear* manner or cannot be described precisely at all. Then, with the means available here, we can not predict the system behavior.

However, in many practical applications, we may assume that the closed-loop system, designed without consideration of disturbances, will *approximately* perform in the expected way, and, in particular, will stabilize the real system. The remaining problem is then the steady state error. It will be overcome in the next section by a PI-controller added to the state feedback.

Other state space methods for improving the disturbance behavior are presented in other articles of this work. They are only mentioned here without details:

- If a disturbance variable can be *measured* (which will only be possible in exceptional cases) and at the same time if its influence on the system is described by the state equations, then a so-called *disturbance feedback* can be designed. It generates a control input *u* counteracting the influence of the disturbance.
- If, however, the disturbance is not accessible to measurement but its characteristic form is roughly known (typically slow, approximately constant disturbances or periodic disturbances of known frequency), then its influence can be modeled by a so-called *disturbance model* consisting of additional homogeneous state differential equations. A *state observer* for the resulting extended system can then be introduced to *estimate the disturbances*. This estimate is used to run a *disturbance feedback*.
- Sometimes a conventional state feedback law can be found such that the influence of a disturbance *z* on *y* can fully be suppressed. The disturbance transfer function $G_z(s)$ equals zero in that case. This situation is called *disturbance decoupling*.

1.2 Model Uncertainty and Parameter Variations

Further factors influencing the behavior of any real system are *uncertainties in the linear system model* and *variations of system parameters* over time. The elements of the matrices $\mathbf{A}, \mathbf{b}, \mathbf{c}^\mathrm{T}$, describing the plants considered here, never will precisely reflect the parameters of the real system, either because they are neither by calculation nor by measurement available precisely, or, because the assumption of linearity is only approximately fulfilled. In addition, the system parameters may change over time. This can happen slowly, for instance the friction coefficient of a bearing, or abrupt, for instance when the loading of a car changes.

Counteracting the influence of such problems and specifying the allowable intervals of system parameters is a goal of *robust control*. It is presented in other topic articles of this work.

In the context of state space design, if we assume the influence of model uncertainty and parameter variations being sufficiently small (so that stability is preserved and dynamic behavior is roughly as expected), the most significant problem remaining is *steady state error*. It can be eliminated by PI-state-feedback control.

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MATLAB applications: The following two web-addresses provide introductory examples on how to use the software package MATLAB for control system design purposes: http://tech.buffalostate.edu/ctm/ and http://www.ee.usyd.edu.au/tutorials_online/matlab/index.html

Biographical Sketch

Boris Lohmann received the Dipl.-Ing. and Dr.-Ing. degrees in electrical engineering from the Technical University of Karlsruhe, Germany, in 1987 and 1991 respectively. From 1987 to 1991 he was with the Fraunhofer Institut (IITB) and with the Institute of Control Systems, Karlsruhe, working in the fields of autonomous vehicles control and multi-variable state space design.

From 1991 to 1997 he was with AEG Electrocom Automation Systems in the development department for postal sorting machines, at last as the head of mechanical development. In 1994 he received the 'Habilitation' degree in the field of system dynamics and control from the Universität der Bundeswehr, Hamburg, for his results on model order reduction of nonlinear dynamic systems.

Since 1997 he has been full professor at the University of Bremen, Germany, and head of the Institute of Automation Systems. His fields of research include nonlinear multivariable control theory; system modeling, simplification, and simulation; and image-based control systems, with industrial applications in the fields of autonomous vehicle navigation, active noise reduction, error detection and fault diagnosis.