ESTIMATION AND COMPENSATION OF NONLINEAR PERTURBATIONS BY DISTURBANCE OBSERVERS

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Contents

Summary

Dynamical systems are often influenced by troublesome nonlinear effects such as Coulomb friction, hysteresis or backlash. In this chapter an indirect measuring technique of the actual values of these nonlinearities is presented. Based on a fictitious model of the time behavior of the nonlinearities a linear state observer of an extended dynamical system is designed resulting in estimates of the nonlinear effects. In the case of control design the disturbance rejection control method is applied to counteract the nonlinearities by the estimated signals. Some sufficient criteria for the asymptotic stability of the estimation error are given. Additionally some applications are mentioned in the fields of high accurate position control of robots and of fault detection of cracks in turbine rotors.

1. Introduction

Nonlinear dynamical systems show a variety of phenomena which are unknown in linear control systems. Different solution behavior depending on the initial conditions, the existence of limit cycles or strange attractors, the appearance of jumps of amplitude , phase or frequency in case of self-excited or forced vibrations are some examples of these phenomena. The variety of nonlinear phenomena has generated a variety of different design methods for nonlinear control systems. Only recently the method of exact linearization and nonlinear system decoupling by state feedback becomes a unifying approach to nonlinear control design. But the method needs a big amount of calculations and, additionally, the smoothness requirements of the nonlinear functions under consideration

Many technical applications do not satisfy these assumptions of the exact linearization method. Mechanical control systems show, for example, discontinuous nonlinear characteristics such as Coulomb friction, hysteresis, or backlash. Typically these effects appear in motor drive control or, more generally, in machine dynamics where the system behavior is predominantly governed by linear differential equations but superposed by additional nonlinear “dirty effects”. For the analysis and design of such systems usually the knowledge of the nonlinear characteristics is required. But often this knowledge is not available, especially if the characteristics change during operation.

Therefore based on the method of disturbance observers a method has been developed to estimate the time behavior and/or the mathematical description of the nonlinear characteristics. Using these estimates of nonlinearities a feedback control can be designed counteracting the influence of the nonlinearities. The design is based on the theory of disturbance rejection. Therefore, by applying linear control theory nonlinear effects will be estimated and compensated leading to satisfactory closed-loop control systems.

2. Problem Statement

We are dealing with control problems which are described in the state space by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{N}\mathbf{f}(\mathbf{x}(t), t) + \mathbf{B}\mathbf{u}(t)\,, \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{2}$$

where $\mathbf{x}$, $\mathbf{u}$, $\mathbf{y}$ denote the *n*-dimensional state vector, the *r*-dimensional control vector and the *m*-dimensional measurement vector, respectively. The vector $\mathbf{f}(\mathbf{x}, t)$ represents *p* more or less unknown functions which are generally nonlinear but may be in special cases linear functions with unknown parameters or external disturbances depending only on time. The matrices , $\mathbf{N}$, , $\mathbf{C}$ are of appropriate dimensions representing the system matrix, nonlinearity, control input matrix, and the measurement matrix, respectively. To avoid redundant formulations the conditions

$$\text{rank}\,\mathbf{N} = p\,,\ \text{rank}\,\mathbf{B} = r\,,\ \text{rank}\,\mathbf{C} = m \tag{3}$$

are assumed to be satisfied.

The first aim will be the construction of an estimate $\hat{\mathbf{f}}(\hat{\mathbf{x}}(t), t)$ of the unknown, nonlinear effects. The second, the control aim, consists in the design of a suitable dynamic output feedback such that the closed-loop control system is asymptotically stable and the *q*-dimensional error vector

$$\mathbf{z}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{u}(t) \tag{4}$$

is controlled independent of the influence of $\mathbf{f}(\mathbf{x},t)$:

$$z(t) \to 0 \text{ for } t \to 0. \tag{5}$$

The control design task is solved in two steps. Firstly an estimate $\hat{\mathbf{f}}(\hat{\mathbf{x}}(t),t)$ will be constructed independent of the feedback control. Secondly, a dynamic output feedback will be designed assuring asymptotic stability as well as the compensation of the nonlinearities in the error vector such that (5) holds.

3. Theory

3.1. Estimation of Nonlinearities

The fundamental idea for the estimation $\hat{\mathbf{f}}(t)$ of $\mathbf{f}(\mathbf{x}(t),t)$ consists in the approximation of its time behavior by some base functions which are solutions of a fictitious linear dynamical system:

$$\mathbf{f}(\mathbf{x}(t),t) \approx \mathbf{H}\mathbf{v}(t), \tag{6}$$

$$\dot{\mathbf{v}}(t) = \mathbf{V}\mathbf{v}(t). \tag{7}$$

The system (7) has to be selected suitably with respect to its dimension *s* as well as to its system matrix **V**. Later it will be shown that often the choice $\mathbf{V} = \mathbf{0}$ is satisfactory. This approximate model (6)-(7) is only used for the design of the linear observer (8); it is not expected to describe the (nonlinear) perturbations exactly by (6)-(7).

Substituting for the nonlinearities of (1) by the model (6)-(7) an extended linear system is obtained for which a state observer can be designed. In the case of an identity observer the estimation system happens to be as follows:

$$\begin{bmatrix} \dot{\hat{\mathbf{x}}} \\ \dot{\hat{\mathbf{v}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{NH} \\ \mathbf{0} & \mathbf{V} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{v}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_v \end{bmatrix} (\mathbf{y} - \hat{\mathbf{y}}) \quad = \begin{bmatrix} \mathbf{A} - \mathbf{L}_x\mathbf{C} & \mathbf{NH} \\ -\mathbf{L}_v\mathbf{C} & \mathbf{V} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{v}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_v \end{bmatrix} \mathbf{y} \tag{8}$$

The choice of the observer gain matrices $\mathrm{L}_{\mathbf{x}}, \mathrm{L}_{\mathbf{v}}$ can be realized such that the observer (8) is asymptotically stable if the extended system is detectable. Moreover, arbitrary eigenvalues can be realized if the extended system is completely observable:

$$\operatorname{rank}\begin{bmatrix} \lambda\mathbf{I}_n - \mathbf{A} & -\mathbf{NH} \\ \mathbf{0} & \lambda\mathbf{I}_s - \mathbf{V} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} = n + s \text{ for all } \lambda \in C. \tag{9}$$

With the estimated signals of (8) the nonlinearities can be reconstructed. Their time behavior is estimated by

$$\hat{\mathbf{f}}(t) = \mathbf{H}\hat{\mathbf{v}}(t). \tag{10}$$

3.1.1. Comments on The Observability Condition

The observability condition (9) includes complete observability of the linear part of the original system (1)-(2), complete observability of the fictitious model (6)-(7) where (6) is considered as an output equation for system (7), and the complete transfer behavior of the modes of (7) to the measurement (2). The last property can be generally assured independent of the choice of the fictitious model if there are no transfer zeros from the nonlinearity input variables to the output variables:

$$\operatorname{rank}\begin{bmatrix} \lambda\mathbf{I}_n - \mathbf{A} & \mathbf{N} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} = n + p \text{ for all } \lambda \in C. \tag{11}$$

Additionally, by (11) it is shown that the number of measurements must be at least equal to the number of nonlinearity inputs:

$$m \geq p. \tag{12}$$

3.1.2. Choice of Fictitious Model

The signals of the fictitious model (6)-(7) should approximate the time behavior of the nonlinearities as closely as possible. A suitable choice of the matrices **H**, **V** requires usually a good *à priori* knowledge of the system behavior. However, in many applications a simple consideration is more efficient.

Consider the true time behavior of the nonlinearities to be approximated by step functions; then we have an approximation by piecewise constant basis functions, i.e. $\mathbf{V} = \mathbf{0}$ piecewise. If the observer (8) is fast enough it will follow the changes of the step function. Therefore, often the choice of the fictitious model can be simply realized by p integrators:

$$s = p, \ \mathbf{H} = \mathbf{I}_p, \ \mathbf{V} = \mathbf{0}. \tag{13}$$

3.1.3. PI-observer

The choice of the fictitious model (6)-(7) according to the proposal (13) leads to a PI-observer, i.e. an observer which feeds back the measurement error as a proportional and integral combination. The observer (8) yields

$$\hat{\mathbf{v}} = -\mathbf{L}_{\mathrm{v}} \int \left(\mathbf{y} - \hat{\mathbf{y}}\right) dt\,, \tag{14}$$

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{L}_{\mathbf{x}} \left(\mathbf{y} - \hat{\mathbf{y}}\right) + \mathbf{N}\mathbf{L}_{\mathrm{v}} \int \left(\mathbf{y} - \hat{\mathbf{y}}\right) dt \tag{15}$$

where $\hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{x}}$ is the estimated measurement and $\mathbf{y} - \hat{\mathbf{y}}$ is the measurement error. The estimated nonlinearities are given by

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{v}}(t)\,. \tag{16}$$

The PI-observer has been shown in many applications as a very efficient estimator of the unknown effects. For a constant disturbance $\mathbf{f}(t) \equiv \mathbf{f}_0 = \text{const.}$ the PI-observer ensures steady-state accuracy.

3.2. Convergence and Estimation Errors

In this section the convergence of the estimate (10) to the true behavior is discussed, i.e., it is checked if the nonlinear perturbation is asymptotically estimated by the linear observer (8):

$$\hat{\mathbf{f}} \to \mathbf{f}(\mathbf{x}(t), t)\,. \tag{17}$$

To simplify the notation we confine ourselves to the integrator model (13). Defining the estimation errors $\mathbf{e}_{\mathbf{x}} = \hat{\mathbf{x}} - \mathbf{x}$, $\mathbf{e}_{\mathbf{f}} = \hat{\mathbf{v}} - \mathbf{f}$ then the error equations of the observer (8) are given by

$$\begin{bmatrix} \dot{\mathbf{e}}_{\mathbf{x}}(t) \\ \dot{\mathbf{e}}_{\mathbf{f}}(t) \end{bmatrix} = \mathbf{A}_{\mathrm{b}} \begin{bmatrix} \mathbf{e}_{\mathbf{x}}(t) \\ \mathbf{e}_{\mathbf{f}}(t) \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \dot{\mathbf{f}} \end{bmatrix} \tag{18}$$

where the observer system matrix $\mathbf{A}_{\mathrm{b}}$ is

$$\mathbf{A}_{\mathrm{b}} = \mathbf{A}_{\mathbf{e}} - \begin{bmatrix} \mathbf{L}_{\mathbf{x}} \\ \mathbf{L}_{\mathrm{v}} \end{bmatrix} \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix},\ \mathbf{A}_{e} = \begin{bmatrix} \mathbf{A} & \mathbf{N} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{19}$$

and the time derivative

$$\dot{\mathbf{f}}(t) = \frac{d}{dt}\mathbf{f}(\mathbf{x}(t), t) \quad = \frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{f}}{\partial \mathbf{x}^{\mathrm{T}}}\left(\mathbf{A}\mathbf{x} + \mathbf{N}\mathbf{f} + \mathbf{B}\mathbf{u}\right) \tag{20}$$

can be given by (7) if $\mathbf{f}(\mathbf{x}, t)$ is differentiable one time at least.

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Biographical Sketch

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1978	Professor of Mechanics
Since 1981	University of Wuppertal Professor of Safety Control Engineering
1983 - 1985	Dean of the Department of Safety Engineering
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1995 – 1999	Vice Rector of the University of Wuppertal
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1988 - 2000	Selection Committee of the Alexander von Humboldt Foundation
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