

## PARAMETER ESTIMATION FOR DIFFERENTIAL EQUATIONS

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### Summary

Nonlinear dynamical systems are usually identified by matching a given input-output behavior with empirical discrete-time approximations such as artificial neural networks, fuzzy models, models based on wavelets (see *Identification of Nonlinear Systems, Nonparametric System Identification, Identification of Block-Oriented Models, Identification of NARMAX and Related Models, System Identification Using Neural Networks, System Identification Using Fuzzy Models, System Identification Using Wavelets, Parameter Estimation for Nonlinear Continuous-Time State-Space Models from Sampled Data, Identification in the Frequency Domain, and Parametric Identification using Sliding Modes*) etc.

The physical values of the parameters cannot be extracted from these formulations. Techniques for dealing with models based on physical laws are applicable to a restricted class of systems and have large computational complexity. In this chapter a methodology is presented that is applicable to a broad class of nonlinear differential equations. This is achieved by making use of a set of modulating functions for characterizing the continuous process signals.

The advantages of modulating functions are that the differential equation is reduced to an algebraic one, the formulation is free from boundary conditions, and the computations can be made using fast algorithms for standard discrete transformations. The resulting estimation scheme is applied to different categories of nonlinear systems and is tested under both noise-free and noisy conditions. This estimation scheme is also applied to an inverted pendulum model.

## 1. Introduction

There are a number of techniques for estimating the parameters of continuous-time systems described by ordinary differential equations (see *Continuous-time Identification*). These involve computation of suitable measures of the derivatives of the input-output data. The use of these measures instead of the original signals leads to an algebraic formulation. These measures are usually multiple time integrals or filtered versions of the signals, or the coefficients in an orthogonal series expansion. Direct computation of the derivatives is avoided by the use of these measures. However, it is not straightforward to apply these methods to systems governed by non-linear differential equations (NDE).

The major difficulty is that an NDE is, in general, not integrable. This makes the integral or simple linear filtering methods inapplicable. Use of a scheme like the extended Kalman filter results in a very complex formulation without guaranteed convergence. Filtering methods can be applied in a straightforward way only if the NDE is exactly integrable, that is, if its terms can be written as pure derivatives of some computable function of the measured signals. This class of systems will be referred to as integrable nonlinear systems.

Another serious difficulty is that of computational burden. Eventhough, in principle, signals can be expanded in terms of continuous orthogonal basis functions, such as Legendre polynomials, evaluation of the coefficients in the series, especially the higher order ones, requires a great deal of computation. Further computations are required, to

obtain the approximations for the derivatives and the various nonlinear product terms. Due to these reasons, use of the standard linear techniques has been mostly restricted to only very special classes of nonlinear systems, such as bilinear systems, Hammerstein systems, etc, all of which belong to the integrable category.

In Chen and Shih (1978), Cheng and Hsu (1982), Jan and Wong (1981), Karanam et al. (1978), Rao (1983) and Wang (1982), piecewise constant orthogonal expansions were applied to the identification of bilinear systems and in Kung and Shih (1986), Chen and Lin (1986) and Rao and Sivakumar (1982) they were applied to some other categories of nonlinear systems.

Applications of orthogonal polynomials have been reported by Liu and Shih (1984), Lee and Chang (1986), Hwang and Chen (1985, 1986), Chou and Horng (1986, 1987) and Wang et al. (1987) for identification of bilinear systems and by Shih and Kung (1985, 1986), Zaman and Jha (1985), Horng and Chou (1987) and Hwang and Shyu (1988) for Hammerstein systems.

An important exception to the above is the method of modulating functions (MF), introduced by Shinbrot (1957) which can be applied to a more general class of nonlinear systems as described below.

Consider a set of smooth functions  $\{\phi_m(t), m = 1, 2, \dots\}$  on a finite interval  $t \in (0, T)$ . Let each function and its time derivatives up to order  $n-1$  vanish at the limits of the interval, that is,

$$\phi_m^i(t) = 0 \text{ for } t = 0 \text{ and } t = T, i = 0, 1, \dots, n-1, \quad \forall m, \quad (1)$$

where  $\phi_m^i(t)$  is the  $i^{\text{th}}$  time-derivative of  $\phi_m(t)$ .

The  $i^{\text{th}}$  derivative  $x^i(t)$  of a function  $x(t)$  defined over the interval  $(0, T)$  is said to be modulated by  $\phi_m(t)$  by the following operation

$$\bar{x}_m^i = \int_0^T \phi_m(t) x^{(i)}(t) dt. \quad (2)$$

Then, by virtue of the property given by Eq. (1), the boundary conditions of  $x^{(i)}(t)$  are eliminated from the modulated component  $\bar{x}_m^i$  for  $i \leq n$ . Mainly, two types of such modulating functions have so far been proposed for the identification of nonlinear systems. The first one was introduced in 1985 by Pearson and Lee who used the Fourier integrating kernels to construct the Fourier Modulating Functions (FMF). The FMF spectra are easy to compute using fast algorithms for discrete Fourier Transform (DFT). Another advantage of this system is the simple relation which exists between the Fourier spectra of a signal and its derivatives.

However, these are complex-valued functions naturally giving rise to a set of complex

equations. When either its real or imaginary components alone are used as the modulating functions, the frequency content of the data is not fully exploited. Therefore, in general, a large amount of computation is required, especially when convolutions have to be performed with these spectral components, as in the case of nonlinear systems. In 1995, Patra and Unbehauen have proposed another class of modulating functions, named as the Hartley Modulating Functions (HMF), which are based on the Hartley transform, and lead to real-valued spectra. Other important references related to modulating functions are Co and Ydstie (1990), Co and Ungarala (1997), Jalili et al. (1992), Preisig and Rippin (1993), Saha and Rao (1983) and Ungarala and Co (2000).

As mentioned above, using modulating functions one can considerably extend the class of nonlinear systems for which an algebraic formulation can be obtained which will be free from the initial conditions. One may consider models described by the following differential equation

$$\sum_{j=0}^{n_1} \sum_{k=1}^{n_2} g_j(\mathcal{G}) F_{jk}(u, y) P_{jk}(p) E_k(u, y) = 0. \quad (3)$$

Here,  $g_j(\mathcal{G})$  is a function of the unknown parameter vector  $\mathcal{G}$ ,  $F_{jk}$  and  $E_k$  are known functions of  $u$  and  $y$  (without any derivatives), and  $P_{jk}(p)$ , with  $p = d/dt$ , is a polynomial in  $p$ . Many physical systems can be expressed in the above form, which would be referred to as the modulatable form of a nonlinear model. With such a description, it is possible to apply the modulating functions approach and arrive at a computationally feasible parameter estimation procedure that minimizes an explicit cost function of the form

$$J(\mathcal{G}) = \sum_{j=0}^{n_1} \sum_{k=0}^{n_2} r_{jk} g_j(\mathcal{G}) g_k(\mathcal{G}) \quad (4)$$

The computation of  $r_{jk}$ , in general, involves complex convolutions. However, if  $F_{jk} = \text{a constant for all } j \text{ and } k$ , then (3) reduces to an integrable NDE, for which no convolutions are necessary.

With the above background, this chapter describes a parameter estimation procedure for nonlinear continuous-time models described by a class of NDE given by (3). The family of Hartley modulating functions (HMFs) is used to illustrate the methodology. The advantages of HMFs are that, they are real valued, and the Hartley spectra can be computed efficiently with the help of fast algorithms for discrete Hartley transformation (DHT). The resulting formulation for parameter estimation is free from boundary conditions and gives rise to linear-in-parameters model.

The chapter is organized as follows. Section 2 gives a brief introduction to continuous and discrete Hartley transformation (HT), whose integrating kernels constitute the HMF. Some important relations and properties of HTs are summarized. In Section 3 the

HMFs are formally defined. Also the formulae necessary for the application of these to the parameter estimation problem are derived. Section 4 describes the formulation of the parameter estimation problem.

For this purpose we consider some typical systems, namely Linear, Integrable and Modulatable nonlinear systems. In Section 5 some of the computational issues involved in obtaining the HMF spectra of signals are discussed. Section 6 illustrates the technique for various categories of nonlinear systems. In Section 7 the methodology is applied to the parameter estimation of an inverted pendulum model. Finally, Section 8 makes some concluding remarks.

## 2. The Hartley Transformation

A brief introduction to Hartley transformation is presented here. Some important definitions and some useful properties are discussed. Further details can be found in Bracewell (1986).

### 2.1. The Continuous Hartley Transform (CHT)

The CHT of a continuous signal  $x(t)$  is defined by the integral

$$H(\omega) = \int_{-\infty}^{\infty} x(t) \text{cas } \omega t \, dt \quad (5)$$

where

$$\text{cas } t = \cos t + \sin t \quad (6)$$

The inverse transform is also given by

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) \text{cas } \omega t \, d\omega. \quad (7)$$

In the above it has been assumed that the signal  $x(t)$  is such that the integral in (5) exists. It is obvious that the transform  $H(\omega)$  is real-valued for a real valued  $x(t)$ . The variable  $\omega$  plays the role of frequency as in the well known Fourier transform and these two transforms are also linearly related to each other. It should be pointed out here that, even though this transform is real-valued, it is a one-to-one transform and does not lead to any loss of information from the frequency content of the data. However, one must utilize both the positive and negative sides of the spectrum to ensure this.

### 2.2. Properties of CHT

Some useful theorems related to CHT are as follows.

#### 2.2.1. Scaling of Variable

For  $T \neq 0$ ,

$$\int_{-\infty}^{\infty} x(t/T) \cos \omega t dt = |T| H(T\omega). \quad (8)$$

### 2.2.2. Convolution in Time-domain

If  $v(t) = x_1(t) * x_2(t)$ , where the operator  $*$  denotes the time domain convolution

$$\int_{-\infty}^{\infty} x_1(t - \tau) x_2(\tau) d\tau,$$

then the CHT of  $v(t)$  is given by

$$H_v(\omega) = \frac{1}{2} [H_1(\omega) H_2(\omega) - H_1(-\omega) H_2(-\omega) + H_1(\omega) H_2(-\omega) + H_1(-\omega) H_2(\omega)], \quad (9)$$

where  $H_1(\omega)$  and  $H_2(\omega)$  are the CHTs of  $x_1(t)$  and  $x_2(t)$  respectively.

### 2.2.3. Multiplication in the Time-Domain

If  $v(t) = x_1(t) x_2(t)$ , then the CHT of  $v(t)$  is given by

$$H_v(\omega) = \frac{1}{2} [H_1(\omega) * H_2(\omega) - H_1(-\omega) * H_2(-\omega) + H_1(\omega) * H_2(-\omega) + H_1(-\omega) * H_2(\omega)], \quad (10)$$

where  $H_1(\omega)$  and  $H_2(\omega)$  are the CHTs of  $x_1(t)$  and  $x_2(t)$  respectively. Note that the right-hand side expression of (10) can also be written as

$$E_1(\omega) * H_2(\omega) + O_1(\omega) * H_2(-\omega),$$

where  $E_1(\omega)$  and  $O_1(\omega)$  denote the even and odd parts of  $H_1(\omega)$  respectively. Because of the frequent occurrence of the above expression, it will be hereinafter denoted by  $H_1(\omega) \otimes H_2(\omega)$ . In general, it can be computed via two convolutions between real functions, although if one of the functions is symmetric, only one convolution is necessary.

It may be pointed out in this context how the use of Hartley transform reduces the computational complexity. Use of Fourier transform in the above relation would require convolutions among complex functions in the frequency domain to compute the equivalent of time-domain multiplications in the above.

### 2.2.4. Differentiation

If

$$v(t) = \frac{d^n x}{dt^n} =: x^{(n)} \quad (11)$$

then the CHT of  $v(t)$  is given by

$$H_v(\omega) = [\text{cas}'(n\pi/2)](\omega)^n H((-1)^n \omega), \quad (12)$$

where  $\text{cas}'t = \cos t - \sin t$ . Denoting the first and second derivatives of  $x$  as  $x'(t)$  and  $x''(t)$  respectively, their CHTs are computed as  $-\omega H(-\omega)$  and  $-\omega^2 H(\omega)$  respectively.

The above relations are very useful for understanding the behavior of the HT. However, for computational purposes, it is necessary to define the discrete Hartley Transform as follows.

### 2.3. The Discrete Hartley Transform (DHT)

The DHT of the sequence  $\{x(KT/N)\}$ ,  $k = 0, 1, \dots, N-1$  is defined as

$$\widehat{H}(l) = \frac{1}{N} \sum_{k=0}^{N-1} x(KT/N) \text{cas } 2\pi lk/N \quad (13)$$

for  $l = 0, 1, \dots, N-1$ . The inverse DHT is given by

$$x(KT/N) = \sum_{l=0}^{N-1} \widehat{H}(l) \text{cas } 2\pi lk/N \quad (14)$$

## 3. The Hartley Modulating Functions

### 3.1. Definition

An element  $\phi_m(t)$  of the set of Hartley modulating functions in time interval  $(0, T)$ , which vanishes at the limits of the interval together with all its derivatives up to order  $n-1$ , is defined by

$$\phi_m(t) = \sum_{j=0}^n (-1)^j \binom{n}{j} \text{cas}(n+m-j)\omega_0 t, \quad 0 \leq t \leq T, \quad \omega_0 = 2\pi/T. \quad (15)$$

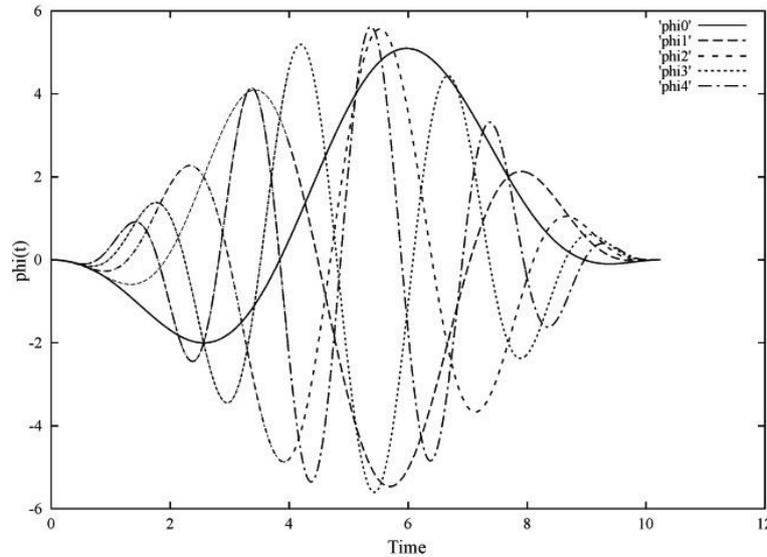


Figure 1: Five members of the family of HMF ( $n = 2$ ).

It can be easily verified that the HMF satisfies the property given by (1). Figure 1 shows five members from the set of Hartley modulating functions of  $n = 2$ . Let us denote the  $m^{th}$  HMF spectral component of a signal  $x(t)$ , defined within the interval  $(0, T)$ , by  $\bar{H}(m\omega_0)$ . That is,

$$\bar{H}(m\omega_0) = \int_0^T x(t)\phi_m(t) dt. \quad (16)$$

Then it can be easily verified by inspection that

$$\bar{H}(m\omega_0) = \sum_{j=0}^n (-1)^j \binom{n}{j} H((n+m-j)\omega_0). \quad (17)$$

where  $H(\omega)$  is the CHT of  $x(t)$ . Therefore, the HMF spectra of a signal can be obtained as linear combinations of the sampled values of the CHT of the signal. The above operation will more specifically be referred to as an  $n^{th}$  order successive difference on  $H(\omega)$ .

Some relations, necessary for the application of the HMF to the parameter estimation problem of nonlinear systems are derived below.

### 3.2. Properties of HMF

#### 3.2.1. Spectra for Derivatives of Signals

If  $x^{(i)}(t)$  denotes the  $i^{th}$  derivative of the signal  $x(t)$ , then the HMF spectra of

$x^{(i)}(t)$  for  $1 \leq i \leq n$ , is given by

$$\bar{H}^i(m\omega_0) = \sum_{j=0}^n (-1)^j \binom{n}{j} [\text{cas}'(i\pi/2)](n+m-j)^i \omega_0^i H((-1)^j(n+m-j)\omega_0) \quad (18)$$

Proof: The proof is straightforward by repeated integration by parts, until all the derivatives are transferred to  $\phi_m(t)$ . Because of the property given by 1) all the boundary terms vanish.

Note that the right-hand side of (18) can be considered to be the  $n^{\text{th}}$  order successive difference of the function

$$\tilde{H}^i(m\omega_0) = [\text{cas}'(i\pi/2)](m\omega_0)^i H((-1)^i m\omega_0) \quad (19)$$

This has a form similar to (12), which is the formula for the CHT of the derivative of a signal. This indicates that for a finite time interval  $(0, T)$ , a similar relationship holds in the difference form. In general, for finite time data records, the spectra of the derivatives would contain polynomial functions of  $\omega$  whose coefficients would be the non-zero initial condition terms. These terms vanish due to the successive difference operation inherent in the computation of HMF spectra.

### 3.2.2. Spectra for the Product of a Measured Signal and the Derivative of Another

In handling nonlinear systems one often encounters terms of the form  $x_1(t)x_2^{(i)}(t)$  where  $x_1(t)$  and  $x_2(t)$  are measurable signals. The HMF spectra for such a product is given by

$$\bar{H}_{1,2}^i(m\omega_0) = H_1(m\omega_0) \otimes \bar{H}_2^i(m\omega_0) \quad (20)$$

Proof: The signal  $x_1(t)$  can be expressed in terms of its infinite Hartley series representation to obtain

$$\bar{H}_{1,2}^i(m\omega_0) = \sum_{l=-\infty}^{\infty} H_1(l\omega_0) \int_0^T x_2^{(i)} \text{cas}(l\omega_0 t) \sum_{j=0}^n (-1)^j \binom{n}{j} \text{cas}((n+m-j)\omega_0 t) dt \quad (21)$$

Next, the identity

$$\text{cas } A \text{cas } B = \frac{1}{2} [\text{cas}(A+B) - \text{cas}(-A-B) + \text{cas}(A-B) + \text{cas}(B-A)] \quad (22)$$

is used to obtain

$$\begin{aligned} \overline{H}_{1,2}^i(m\omega_0) = & \sum_{l=-\infty}^{\infty} H_1(l\omega_0) \sum_{j=0}^n (-1)^j \binom{n}{j} \\ & \times \int_0^T x_2^{(i)} \frac{1}{2} \left( \begin{aligned} & \text{cas}((l+m+n-j)\omega_0 t) - \text{cas}((-l-m-n+j)\omega_0 t) \\ & + \text{cas}((l-m-n+j)\omega_0 t) + \text{cas}((-l+m+n-j)\omega_0 t) \end{aligned} \right) dt \end{aligned} \quad (23)$$

Then, applying the method of integration by parts to compute the integral on the right-hand side

$$\begin{aligned} \overline{H}_{1,2}^i(m\omega_0) = & \frac{1}{2} \sum_{l=-\infty}^{\infty} H_1(l\omega_0) \sum_{j=0}^n (-1)^j \binom{n}{j} \text{cas}'(i\pi/2) \\ & \left( \begin{aligned} & (l+m+n-j)^i \omega_0^i H_2 \left( (-1)^i (l+m+n-j)\omega_0 \right) \\ & - (-l-m-n+j)^i \omega_0^i H_2 \left( (-1)^i (-l-m-n+j)\omega_0 \right) \\ & + (l-m-n+j)^i \omega_0^i H_2 \left( (-1)^i (l-m-n+j)\omega_0 \right) \\ & + (-l+m+n-j)^i \omega_0^i H_2 \left( (-1)^i (-l+m+n-j)\omega_0 \right) \end{aligned} \right) \end{aligned} \quad (24)$$

From the definition of  $\widetilde{H}^i$  and the operation  $\otimes$ , the right-hand side in the above expression can be shown to be equivalent to

$$H_1(m\omega_0) \otimes \sum_{j=0}^n (-1)^j \binom{n}{j} \widetilde{H}_2^i((n+m-j)\omega_0) \quad (25)$$

which completes the proof.

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### Biographical Sketch

**Amit Patra** received the B.Tech., M.Tech. and Ph.D. degrees from the Indian Institute of Technology, Kharagpur in 1984, 1986 and 1990 respectively. During 1992-93, and again in 2000, he visited the Ruhr-University, Bochum, Germany as a Post-Doctoral Fellow of the Alexander von Humboldt Foundation. He Joined the Department of Electrical Engineering, Indian Institute of Technology, Kharagpur in 1987 as a faculty member, and is currently a Professor. His research interests include discrete-event and hybrid systems, fault detection and fault tolerant control of industrial processes, power electronics and VLSI Design and industrial automation and control.

He has taught courses on Electrical Circuits, Microprocessors, Real-time Systems, Digital Signal Processing, Data Communication Systems, Stochastic Processes, Estimation and Identification, Optimal Control, Adaptive Control, etc. He has also delivered a number of invited lectures in various Specialized Courses for the Industry. He has published about 80 research papers and is the co-author (with Ganti Prasada Rao) of a research monograph entitled General Hybrid Orthogonal Functions and Their Applications in Systems and Control, Published by the Springer Verlag in 1996.

He has carried out a number of sponsored projects in the areas of fault detection and diagnosis and fault tolerant control. He is currently the Principal Investigator in the project entitled Fault Tolerant Control of Satellite Launch Vehicles sponsored by the Indian Space Research Organisation. He is also involved with two inter-disciplinary projects on Design of Building Automation Systems and Multi-sensor-based Condition Monitoring of Manufacturing Processes, sponsored by the Ministry of Human Resource Development and Department of Information Technology, Govt. of India, respectively. He is a consultant to the National Semiconductor Corporation, USA in the area of Integrated Circuits for Power Management, and to the Calcutta Electric Supply Corporation on Economic Generation Scheduling. Earlier he served in a similar capacity to the Naval Science and Technological Laboratory, Visakhapatnam, Tata Infotech Limited, Mumbai, the Department of Posts Ministry of Communications, Govt. of India and Hughes Escorts Communications Limited, Gurgaon, New Delhi. He is also one of the founding Directors of the Faculty Entrepreneur-ship Company Control and Automation Technologies, Systems and Software, registered with the Science and Technology Entrepreneurship Park, at I.I.T. Kharagpur, and engaged in the development of indigenous technology for industrial automation and

laboratories for electrical engineering education. He served as the Convener, Local Organising Committee for the International Conference on Energy, Automation and Information Technology (EAIT- 2001) held at IIT Kharagpur in December 2001.

Dr.Patra received the Young Engineer Award of the Indian National Academy of Engineering in 1996 and the Young Teachers' Career Award from the All India Council for Technical Education in 1995. He has been a Young Associate of the Indian Academy of Sciences during 1992-97. He is a member of IEEE (USA), Institution of Engineers (India) and a life member of the Systems Society of India. His biography has been included in several Editions of Marquis' Who's Who and the ABI International Directory of Distinguished Leadership.

He is a member of the Editorial Advisory Panel of the International Journal of Electrical Engineering Education.

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SAMPLE CHAPTERS