# PARAMETRIC IDENTIFICATION USING SLIDING MODES

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## Contents



## Summary

An algorithm for parameter identification in continuous time is presented. This methodology is designed for nonlinear uncertain systems with partial measurement of the state vector. Non-measurable states are restored through an observer, designed using variable structure theory. Regarding the specific properties of the latter theory used for the design of the observer (convergence in a finite time, invariance and chattering properties), it is possible to design simultaneously a parameter estimation law, which converges asymptotically to the nominal values of the parameters. Some simulation results highlighting the robustness in face of parameter uncertainties and measurement noise illustrate the use of the identification algorithm.

## 1. Introduction

This chapter investigates the issue of simultaneous identification of the vector state and parameter values of nonlinear systems. Parameter estimation in the linear context is widely applied, especially in industrial plants. The most famous method for linear identification is based on the least squares algorithm, (see *Least squares and instrumental variable methods*). Unfortunately, most of the plants encountered in practice belong to nonlinear systems, which can only be adequately described by nonlinear models. Therefore several recent studies focus on the identification of nonlinear plants but this area has not yet received a huge exposure. This is probably due to the difficulty of designing identification algorithms that could be applied to a reasonably large class of nonlinear systems. One possible solution to nonlinear identification consists to use again the least squares algorithm methods. However, this implies a linearization around some equilibrium point which restricts a lot the range of the study. Moreover, other techniques for nonlinear identification have already been investigated such as Volterra-series approach, (see *Volterra and Fliess series expansions*) or the method of maximum-likelihood, (see *Practical issues of system identification*). Nevertheless, both of the previous methods are designed for specific nonlinear systems and are assuming that all the vector state is measurable, which sometimes could be impossible to realize in practice. Therefore our main goal here is to introduce a parameter identification law for a large class of nonlinear systems for which not all the states are measurable. This approach combines parameter identification with estimation of non-measurable states restored by using an observer. In order to do this, we generalize to nonlinear parameter identification the well known work on linear parameter identification based on variable structure theory (see *Sliding mode control*). In classical adaptive control, an additional property (the persistent excitation) is required in order to ensure the parameter convergence. In our approach, the use of the well-known chattering property, an inherent property of the variable structure observer, makes possible the design of the parameter estimation law, even in a finite time, without having to add additional properties.

In Section 2, a specific sliding observer for linear analytic nonlinear systems where only the first state is measurable, is proposed. Thanks to the use of a Lyapunov candidate function, one can prove its convergence in finite time. In Section 3, we introduce a new parameter estimation law. The convergence of the algorithm is proving by using the invariance and chattering properties of the sliding regimes. However, this new parameter estimation law is restricted to nonlinear systems where the control part is linear. In Section 4, the parameter vector and the state vector are derived without the use of a persistent signal excitation. Simulations results in Section 5 illustrate the use of the algorithm and highlight that the method is robust with respect to parameter uncertainties and additive measurement noise.



## 2. State Identification

The aim of this section is to derive an observer for the state vector of a linear analytic nonlinear system of the form

$$\dot{x} = A(x,t) + B(x,t)u$$

First, for this class of systems, it is possible to determine a controllability form which could be represented by a triangular system as follows, (see *Design of nonlinear control systems*)

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\quad \cdots \\ &\dot{x}_n = f(x) + g(x)u \\ &y = x_1 \end{aligned} \qquad (1)$$

with $x \in R^n$ Without loss of generality, let us assume that the measurable state is $x_1$. Moreover, assume that $f(x)$ et $g(x)$ are Lipschitz, that (1) is input to state stable and that the input *u* is bounded. Let us note by $\hat{x}$ the estimation of the vector state (1). The dynamics of each component $\hat{x}_i, i = 1,...,n$ of $\hat{x}$ are designed such that

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + v_1 \\
&\cdots \\
\dot{\hat{x}}_i &= \hat{x}_{i+1} + v_i \\
&\cdots \\
\dot{\hat{x}}_n &= f(\hat{x}) + g(\hat{x})u + v_n
\end{aligned} \tag{2}$$

With

$$\begin{aligned}
v_1 &= k_1 \operatorname{sgn}(x_1 - \hat{x}_1) \\
v_i &= k_i \operatorname{sgn}(z_i)
\end{aligned}$$

$$\tau_i \dot{z}_i + z_i = v_{i-1} \text{ for } i = 2,3,...,n \tag{3}$$

From (1) and (2), the dynamics of the state estimation errors are given by

$$\begin{aligned}
e_1 &= x_1 - \hat{x}_1 \\
\dot{e}_1 &= e_2 - k_1 \operatorname{sgn}(e_1) \\
e_i &= x_i - \hat{x}_i
\end{aligned} \tag{4}$$

$$\dot{e}_i = e_{i+1} - v_i \text{ for } i = 2,3,...n-1 \tag{5}$$

$$\begin{aligned}
e_n &= x_n - \hat{x}_n \\
\dot{e}_n &= f(x) - f(\hat{x}) + g(x)u - g(\hat{x})u - v_n
\end{aligned} \tag{6}$$

The following result can be proved: *There exists a compact subset of* $R^n$*, denoted* $\mathcal{D}$*, such that, for any initial condition* $x_0 \in \mathcal{D}$*, we can find positive constants* $k_i, i = 1,...,n$ *sufficiently large and positive constants* $\tau_i, i = 2,...,n$ *sufficiently small such that the state estimation errors* $e_i, i = 1,...,n$ (4), (5), (6) *converge to zero in a finite time.*

**Proof**

*First Step*: Let us first consider the candidate Lyapunov function

$$V_1 = \frac{1}{2} e_1^2$$

Taking into account (4), its derivative with respect to time can be written

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(e_2 - k_1 \operatorname{sgn}(e_1)) \text{ and} \\ \dot{V}_1 = e_1 e_2 - k_1 |e_1| \tag{7}$$

Therefore,

$$\dot{V}_1 \le |e_1||e_2|_{\max} - k_1|e_1| \tag{8}$$

Finally, if the positive constant $k_1$ is chosen such that

$$k_1 > |e_2|_{\max}$$

$\dot{V}_1$ will be negative definite. A sliding mode is reached after a finite time, $t_1$. The sliding manifold is defined by

$$e_1 = 0; k_1 > |e_2|_{\max}$$

Moreover, thanks to the invariance property, the state estimation error $e_1$ and its time derivative are such that

$$e_1(t) = \dot{e}_1(t) = 0,\ \ \forall t \ge t_1 .$$

Finally, there exists a finite time $t_1$ and a constant $k_1$ such that

$$\forall t \ge t_1 \begin{cases} \hat{x}_1(t) = x_1(t) \\ \dot{e}_1(t) = 0 \end{cases} \tag{9}$$

*Step i for* $i = 2,3,...,n-1$ Let us study now the following candidate Lyapunov function

$$V_i = \frac{1}{2}e_i^2 + V_{i-1}$$

By using the previous steps and (9), the time derivative is given by

$$\dot{V}_i \le e_i \dot{e}_i$$

From (5) we can write

$$\dot{V}_i \le e_i(e_{i+1} - v_i) \text{ and} \\ V_i \le e_i(e_{i+1} - k_i \operatorname{sgn}(e_i)) \tag{10}$$

Let us now write again Eqs. (10); it follows that

$$\dot{V}_i \leq |e_i| |e_{i+1}|_{\max} - k_i |e_i| \tag{11}$$

Therefore, if a positive constant $k_i$ is chosen such that

$$k_i > |e_{i+1}|_{\max}$$

$\dot{V}_i$ will be negative definite. A sliding mode is then reached after a finite time $t_i > t_{i-1}$. The sliding manifold is now defined by

$$e_i = 0; k_i > |e_{i+1}|_{\max}$$

Moreover, taking into account the invariance conditions, the state estimation error $e_i$ and its time derivative are such that

$$e_i(t) = \dot{e}_i(t) = 0, \ \forall t \geq t_i$$

Finally, there exists a finite time $t_i$ and a constant $k_i$ such that

$$\forall t \geq t_i \begin{cases} \hat{x}_i(t) = x_i(t) \\ \dot{e}_i(t) = 0 \end{cases} \tag{12}$$

*Step n* : Let us consider the Lyapunov function:

$$V_n = \frac{1}{2} e_n^2 + V_{n-1}$$

By considering the previous steps and (12), the following inequality of the time derivative of this Lyapunov function can be written

$$\dot{V}_n \leq e_n \dot{e}_n$$

Therefore, by taking into account (6), we obtain

$$\begin{aligned} &\dot{V}_n \leq e_n (f(x) - f(\hat{x}) + g(x)u - g(\hat{x})u - v_n) \ and \\ &\dot{V}_n \leq e_n (f(x) - f(\hat{x}) + g(x)u - g(\hat{x})u) - e_n (k_n \operatorname{sgn}(e_n)) \end{aligned} \tag{13}$$

for $k_n$ sufficiently small. Now from the hypothesis of the system (1), Eq. (13) can be written

$$\dot{V}_n \leq |e_n| (\alpha |e_n| - k_n)$$

with $\alpha$ as a Lipschitz constant. Therefore, if the constant $k_n$ is chosen such that

$$k_n > \alpha |e_n|$$

then $\dot{V}_n$ will be negative definite. A sliding mode is reached after a finite time, denoted $t_n > t_{n-1}$ The sliding manifold is defined by

$$e_n = 0;\ k_n > \alpha |e_n|$$

By using the invariance conditions, the state estimation error $e_n$ and its time derivative satisfy

$$e_n(t) = \dot{e}_n(t) = 0,\ \forall t \geq t_n$$

and we can see that there exist a finite time $t_n$ and a positive constant $k_n$ such that

$$\forall t \geq t_n \begin{cases} \hat{x}_n = x_n \\ \dot{e}_n = 0 \end{cases} \tag{14}$$

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**Biographical Sketches**

**Fabienne Floret** was born in Avignon (France), in 1975. She received a degree in Electrical Engineering from the University of Cergy in 1998. She has recently obtained her Ph.D thesis at theUniversitÈ of Paris-Sud, SUPELEC. Her main interests and the topics of her thesis are identification, observation, and control of nonlinear and non-linearly parameterized systems.

**Francoise Lamnabhi-Lagarrigue** is Directeur de Recherche au Centre National de la Recherche Scientifique since 1993. She obtained the "maitrise es sciences mathematiques pures" degree at the Universite Paul Sabatier (Toulouse) in 1976 and she held a CNRS position in 1980. She obtained her Docteur d'Etat es Sciences Physiques from Universite Paris Sud in 1985. Some recent activities :

- Editor of International Journal of Control;
- Responsible (1998-2001) of an agreement with the Groupe PSA on the Global Chassis Control project;
- Coordinator (1998-2002) of the Nonlinear Control Network (NCN)

  http://www.supelec.fr/lss/NCN;
- Director of the Marie Curie Control Training Site (CTS) http://www.supelec.fr/lss/CTS;
- Expert evaluator at the European Commission;
- Nominated member of the Board of Governors of the IEEE Control Systems Society for the year 2002 and elected Member for 2003-2005.

Her main research interests lie in the fields of nonlinear systems including system analysis and control design. Several important contributions should be emphasized. The analysis of nonlinear systems using functional expansions based on the Fliess generating power series and the development of formal computing for the Volterra series expansion associated with the output of nonlinear systems. Then singular optimal control and singular tracking for nonlinear systems have been studied. In the same time, an exact combinatorial formula for the derivatives of the output in terms of the derivatives of the inputs has been derived. More recently, her research interests include performance and robustness issues in nonlinear control, identification of nonlinear systems, and the application of these techniques to power systems, to vehicle global chassis control, to magnetic suspension, and to hydraulic actuators.

Francoise Lamnabhi-Lagarrigue is author and co-author of more than 100 publications which include one book, 50 journal papers and chapters of book, a Special issue of the International Journal on Control (Recent advances in the control of nonlinear systems, Vol 71(5), 1998), the co-edition of 6 books (the more recent ones being: - Stability and Stabilization of Nonlinear Systems, LNCIS 246, Springer-Verlag, 1999; - Nonlinear Control in the Year 2000, LNCIS 258 et LNCIS 259, Springer-Verlag, 2000; - Advances in Control of Nonlinear Systems, LNCIS 264, Springer-Verlag, 2001; - Commande des Systemes Non Lineaires, 2 volumes, Traite IC2, Hermes, 2002), about 45 published conference papers, 2 theses and various reports and European proposals.