

## ADAPTIVE PREDICTIVE CONTROL

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**Keywords:** Predictive control, Adaptive control, Discrete-time systems, Generalized predictive control, Mean-level control, dead-beat control, Robustness, Self-tuning, Identification, Recursive-least-squares estimator, Forgetting-factor, Observer filter, Estimator filter.

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### Summary

A modern approach to self-tuning and adaptive control is to couple a robust parameter estimator to a robust control algorithm. For this purpose long-range prediction based control schemes are known to have advantageous properties for coping with sensitive dead-time estimation, nonminimum-phase and unstable systems. In this work the special case of long-range prediction based

Generalized Predictive control (GPC) is used to point out useful robustness properties which are determined by the cost-function parameters and a pre-specified observer polynomial  $T(q^{-1})$ . The GPC method shares the application properties of other well known Model-Based Predictive Control (MBPC) laws such as Identification and Command (IDCOM) and Dynamic Matrix Control (DMC) and can therefore represent a wide class of linear predictive controllers.

The results are particularly simple for mean-level and state dead-beat objectives. It is shown that the simplest choice of horizons (giving 'mean-level' control) provides dead-beat disturbance rejection. For adaptive control a Recursive-Least-Squares (RLS) estimator is considered for generating the model parameters.

## 1. Introduction

Most nontrivial control problems involve uncertainty. Signals are corrupted by sensor noise and by quantization in analog to digital converters. The plant output depends not only on the control input but also on load-disturbances which are mostly unmeasurable and against which the control law has to regulate. Nonlinearities such as saturation, backlash and stiction in the actuator can affect the controlled response yet are difficult to quantify. Moreover, the stability and performance of the closed-loop depends crucially on one major source of uncertainty – the dynamic behavior of the plant itself. The traditional solution for the vast majority of loops in industry has been to use standard manually-tuned three-term regulators. Good results are obtainable from PID algorithms provided that the plant engineer is familiar with tuning methods, though the procedure can be time consuming, and retuning may be necessary if the dynamics change.

The advent of microprocessors in the '70s stimulated interest in self-tuning algorithms in which the controller settings are adjusted automatically, based on models of process behavior deduced from observations of input/output data. The conceptual structure of a self-tuner is shown in Fig 1. Two time-scales are in operation: the fast feedback controller (generally linear) and a slow outer 'updating' loop which provides the plant model and deduces the controller settings via an analytic design procedure so that: an algorithm is said to have the *self-tuning property* if, as the number of samples approaches infinity, the controller parameters tend to those corresponding to an exactly known plant model.

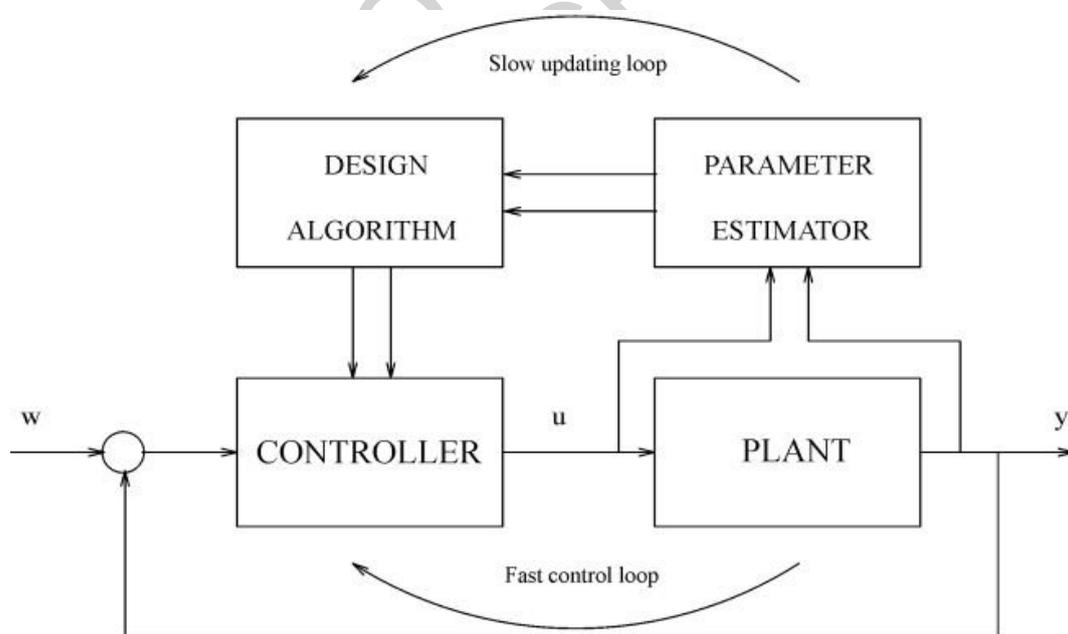


Figure 1: Structure of a general self-tuning controller

Self-tuning theory generally assumes a *constant* plant; in practice the parameter estimator can be designed to update slowly the model parameters, leading to adaptive control.

Many commercial self-tuners simply adjust PID settings, but we are concerned here with a more general-purpose law which is in some sense optimal when used with processes which have complex (possibly time-varying) dynamics such as dead-time and which can include tuned feed-forward terms to cope with measurable load-disturbances. These self-tuners may have several design parameters which can be chosen on-site to tailor the loop behavior according to the plant and design objectives. It is said that these controllers have *design-oriented knobs* as the user is prescribing the character of the closed-loop rather than the  $K$ ,  $T_I$  and  $T_D$  of the PID law. This operational flexibility carries with it the burden of a significant number of prior parameters, so one objective is to ensure either that their selection is easily understood or that the suboptimal performance resulting from bad choices is still acceptable.

Early self-tuning approaches such as the Minimum Variance (MV) and Generalized Minimum Variance (GMV) methods were based on *short-range* prediction of the plant output at a single future instant taking place at the end of the prediction horizon. The length of the prediction horizon was given by the plant's dead-time, and the control was chosen so as to make the projected output equal to the set-point. These are special types of Smith predictor for which the future disturbance at the same time instant is also predicted. Coupled with a recursive least-squares estimator the designs are simple and effective. Their drawback however is the relative sensitivity to incorrectly assumed values of dead-time; with MV in particular the sample interval needs to be chosen with care to avoid instability. Other classical adaptive algorithms have been shown to be sensitive to incorrectly selected model order (i.e. Pole-placement self-tuners) or lack robustness when operating a system with nonminimum-phase behavior.

Because of the inability of classical adaptive control schemes to cope with many applications, adaptive control schemes based on long-range predictions have been introduced. The essential advantage of long-range predictive controllers is that they perform stable control of the majority of real processes and can cope with parameter uncertainty problems using advanced adaptive parameter adjusting techniques. Is specific the plant can be:

1. a nonminimum-phase plant (most continuous-time transfer functions tend to exhibit discrete-time zeros outside the unit circle when sampling rate is fast enough)
2. a plant with variable or unknown dead-time
3. a plant with unknown order
4. an open-loop unstable plant or plant with badly-damped poles.

Methods based on long-range predictions include the following:

- Generalized Predictive Control (GPC)
- Dynamic Matrix Control (DMC)
- IDentification and COMmand (IDCOM)
- Extended Horizon Adaptive Control (EHAC)
- Extended Predictive Self-Adaptive Control (EPSAC)

- Multistep Multivariable Adaptive Control (MUSMAR)
- MULTipredictor Receding Horizon Adaptive Control (MURHAC)

All these methods have certain features in common which distinguish them from previous design philosophies. The solution of a finite horizon optimization problem at each time instant is implemented in a receding horizon way (will be defined later) and the provision of a small number of design parameters allows to select appropriate closed-loop process dynamics. Due to the high quality control performances and very short response time that are often required in modern industrial technology Long-Range Model Based Predictive Control (LRPC/MBPC) implemented on demanding applications have been proven to outperform the classical control schemes in many cases. The importance of LRPC/MBPC is that not only can it handle Multiple-Input/Multiple-Output (MIMO) problems based on experimentally obtained models but also treat the critical matter of input and output constraints in a uniform manner. Many LRPC/MBPC schemes have been applied successfully in process control, electric drives, aircraft and missile control, metallurgical processes, satellite attitude control and navigational course control of ships, just to mention a few practical examples.

Chemical engineers often obtain dynamic information from step tests, or sometimes estimate the related pulse-response model using pseudo-random test signals and cross-correlation. Such models were used for the first time in predictive control in the mid '60s, and at the end of the '70s and were effectively implemented on demanding applications. Models based on general difference-equation rather than a step-response model have been used in many MBPC approaches, however the main design objectives of both approaches are very similar, although approached via a different historical route. In this article the Dynamic Matrix Control (DMC) and Generalized Predictive control (GPC) methods, based on a step-response model and a difference-equation model respectively, will be used for a comparative analysis of important aspects involved in applying adaptive LRPC/MBPC schemes. The GPC uses an observer polynomial in its predictions to get 'load-disturbance tailoring' and increased robustness. This high flexibility of the GPC scheme will be used to point out the basic idea and important design-options this control scheme offers to the control engineer. Differences between GPC and DMC will be highlighted and the effectiveness of the adaptive MBPC design procedure is demonstrated using simulation results. It should be noted that this paper only considers single input-output systems, but most results can be extended for the multiple input-output system case.

The structure of this work is as follows: Important aspects of selecting and using different models for predictive control are discussed in Section 2. Section 3 introduces the main idea behind GPC, flexibility in its design and load-disturbance responses. In Section 4 influences of modeling errors are investigated in a robustness analysis. Self-tuning aspects of combining MBPC with a parameter estimator are dealt with in Section 5. The work concludes with some final remarks.

## **2. System models and long-range prediction**

The idea behind MBPC is based on predicting the system response subject to a future control strategy to be optimized. For this purpose a model must be accessible which can

characterize the plant behavior in terms of a finite set of parameters. The major difference between the various MBPC approaches arises from the properties of models selected for each case. Proper choice of model structure is vitally important in the design of practical self-tuners. In particular it is essential to capture dead-time and disturbance dynamics, as plants are characterized by:

**nonlinearity:** models are generally local-linearizations

**load disturbances:** often random steps or Brownian motion

complex dynamics: there may be high-frequency modes which are not represented in a low-order model and can influence the performance of adaptive controllers.

A model can be thought of having two aspects: its *structure*  $\mathcal{M}$  and its actual *parameter* set  $\theta$ . The derivation and implementation of a LRPC/MBPC algorithm depends on the assumed structure; a ‘good’ design gives satisfactory answers to the following questions:

- Can  $\mathcal{M}$  represent a very general class of plants? For example, can it deal with dead-time, unstable, lightly-damped, high-order systems simply by changes in parameter values?
- Is the number of parameters minimal with  $\mathcal{M}$  still giving adequate predictions?
- Can prior knowledge be easily incorporated?
- Is there a realistic model of load-disturbances?

A good model will give better performance; for example the assumption of constant non-zero or of Brownian-motion disturbances automatically leads to the imposition of an integrator into the control law.

Many model structures are used in MBPC: impulse-response, step-response, state-space, transfer function etc.; we shall derive some ideas using a general *infinite impulse-response* before specializing to a model based on *difference-equations* used by GPC.

## 2.1. General long-range prediction models

Consider the general linear model assumed about the plant:

$$y(t) = M(q^{-1})u(t) + N(q^{-1})\xi(t), \quad (1)$$

where  $q^{-1}$  is the backward-shift operator and  $M, N$  are infinite polynomials giving the control and disturbance dynamics. The disturbance is hence described as an uncorrelated sequence  $\{\xi(t)\}$  driving the transfer-function  $N(q^{-1})$ . To obtain a  $j$ -step-ahead predictor we first decompose  $N$  as:

$$N(q^{-1}) = N_j^*(q^{-1}) + q^{-j}N_j(q^{-1}), \quad (2)$$

so that the future output can be written:

$$y(t+j) = q^j M(q^{-1})u(t) + N_j^*(q^{-1})\xi(t+j) + N_j(q^{-1})\xi(t).$$

The first disturbance term is comprised of only future uncorrelated components, whereas the second term involves  $\xi(t)$  which can be reconstructed, using Eq.(1), from known data by:

$$\xi(t) = \frac{y(t) - M(q^{-1})u(t)}{N(q^{-1})}, \quad (3)$$

so that a *minimum-variance* predictor is:

$$\hat{y}(t+j|t) = M(q^{-1})u(t+j) + N_j(q^{-1}) \frac{y(t) - M(q^{-1})u(t)}{N(q^{-1})} \quad (4)$$

$$= \frac{N_j^*(q^{-1})}{N(q^{-1})} M(q^{-1})u(t+j) + \frac{N_j(q^{-1})}{N(q^{-1})} y(t). \quad (5)$$

Note that the assumption of a nontrivial disturbance model leads to the use of output measurements as well as control signals in the prediction. From the definition of  $N_j$  and  $N_j^*$  and dividing Eq.(2) by  $N$  we have:

$$1 = \frac{N_j^*(q^{-1})}{N(q^{-1})} + q^{-j} \frac{N_j(q^{-1})}{N(q^{-1})}. \quad (6)$$

The second term of the RHS has a leading shift operation of  $q^{-j}$  which implies (by comparing powers of  $q^{-i}$  on each side) that the first  $j$  terms of the infinite polynomial  $N_j^*/N$  are  $1 + 0q^{-1} + \dots$ . This means that the component of output prediction dependent on future controls is not affected by the particular  $N(q^{-1})$  assumed. The noise model only affects the prediction of the plant which depends on initial conditions (*free response*) and not on possible future controls.

## 2.2. Dynamic matrix control prediction model

Consider the use of the above prediction equation for the DMC case which has a Brownian motion (random step) noise model:

$$y(t) = M(q^{-1})u(t) + \frac{\xi(t)}{\Delta}, \quad (7)$$

where  $\Delta$  is the differencing operator  $1 - q^{-1}$ . Expansion into polynomial form gives:

$$N(q^{-1}) = 1 + q^{-1} + q^{-2} + \dots$$

which according to Eq.(2) results in

$$N_j(q^{-1}) = 1 + q^{-1} + q^{-2} + \dots \quad \text{and} \quad N_j^*(q^{-1}) = 1 + q^{-1} + q^{-2} + \dots + q^{-j+1}.$$

Taking

$$\frac{N_j(q^{-1})}{N(q^{-1})} = 1 \quad \text{and} \quad \frac{N_j^*(q^{-1})}{N(q^{-1})} = \frac{1 + q^{-1} + q^{-2} + \dots + q^{-j+1}}{1 + q^{-1} + q^{-2} + \dots} = 1 - q^{-j}$$

into account the optimal DMC predictor is:

$$\hat{y}(t + j | t) = M(q^{-1})u(t + j) - M(q^{-1})u(t) + y(t).$$

It is interesting to note that  $y(t) - Mu(t)$  is the difference between the actual output measurement and the predicted outcome from past inputs. The DMC equations are usually described in terms of control ‘moves’  $\Delta u(t)$  and a step-response model  $S(q^{-1})$ . This can be achieved by using the relations:

$$s_i = s_{i-1} + m_i \quad \text{and} \quad u(t) = \Delta u(t) + \Delta u(t-1) + \dots$$

to give:

$$\hat{y}(t + j | t) = y(t) + s_0 \Delta u(t + j) + s_1 \Delta u(t + j - 1) + \dots + s_j \Delta u(t) + \dots - s_0 \Delta u(t) - s_1 \Delta u(t - 1) - \dots.$$

Noting that  $s_0 = 0$  for proper plants and that  $s_r = K$  where  $K$  is the *dc* plant gain for  $r$  greater than the settling time  $N_s$ , the potentially infinite series in past values of  $\Delta u(t - i)$  become just a finite sum so that:

$$\hat{y}(t + j | t) = \sum_{i=1}^j s_i \Delta u(t + j - i) + y(t) + \sum_{i=1}^{N_s} (s_{i+j} - s_i) \Delta u(t - i). \quad (8)$$

The first part of the RHS is the *forced response* due to a set of hypothetical future controls, the other terms being the *free response* determined by the model structure and currently available data.

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### **Biographical Sketches**

**David William Clarke** gained his doctorate at Oxford University in 1967 for his development of Generalized Least-Squares for system identification. From 1969 he was a University Lecturer at the Department of Engineering Science and a Fellow of New College, Oxford. In 1992 he became Professor of Control Engineering at Oxford and was Head of Department from 1994-1999.

Since 1973 he has been interested in adaptive and self-tuning controls. In 1975 together with P. J. Gawthrop he developed the 'Generalized Minimum-Variance' algorithm. Also in 1975 he built the world's first portable microprocessor-based self-tuner 'SESAME' which was used to investigate the practical use of self-tuning in process industries.

From 1985 he and several coworkers developed Generalized Predictive Control and its application to industrial processes and robotics. His research on adaptive control is concerned with the development of robust stable multivariable self-tuners based on long-range predictive methods for processes subject to state constraints. Increasingly his work has become focused on sensor, actuator and loop validation (SEVA) – a topic enjoying strong international industrial collaboration. His 'Limp-Home Control' – where a multivariable process is reconfigured after instrument failure- uses methods drawn from both predictive control and self-validation.

**Ulfur Ron Halldorsson** was born 18th of May 1970 in Reykjavik, Iceland. He obtained his C. S.-Degree in Electrical Engineering from University of Iceland, in 1994. In Germany he received his Diploma in Electrical Engineering from Technical University of Karlsruhe in 1996 and his Dr.-Ing. Degree in Electrical Engineering from Ruhr-Universität Bochum in 2002. Currently, he is working at the Institute of Automation and Computer Control, Ruhr-Universität Bochum. His current research interests are in nonlinear predictive control.