MODELING OF HYBRID SYSTEMS

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Contents

Summary

The distinguishing characteristic of hybrid systems is the interaction between a continuous-time and a discrete-event component. By modeling these different

components using differential equations and finite state automata, it is possible to represent a wide range of phenomena present in physical and technological systems. This chapter illustrates hybrid dynamics by several simple examples. Some of these examples illustrate properties of hybrid systems not present in purely continuous or purely discrete systems, while others illustrate application domains such as vehicle control and real-time systems. A mathematical model called a hybrid automaton is then introduced, to show how hybrid dynamics can be formally analyzed.

1. Introduction

In the literature, the term "hybrid systems" is used to describe a very wide class of dynamical systems that involve the interaction of heterogeneous data types and dynamics. Of great interest is the class of hybrid systems that arises out of the interaction of continuous dynamics that describe the evolution of a continuous state under differential or difference equations, with discrete dynamics, that describe the evolution of a finite state under automata or other models of computation. This class of hybrid systems has been the focus of intense research activity in recent years. The reason is that it provides a convenient framework for modeling a wide range of engineering systems. For example, the hybrid framework is ideal for modeling systems with multiple time scales, where the fast dynamics can be abstracted away and be treated as discrete changes affecting the slower dynamics. Examples include mechanical systems with collisions, circuits with diodes and switches, chemical processes controlled by valves or pumps, and, most importantly, embedded computation systems, where digital devices interact with an analogue environment.

Another reason for the popularity of hybrid systems is their importance in applications. Methods and tools developed for hybrid systems have already proved useful in a wide range of technological application. Following early work on the verification of digital circuits, the hybrid formalism and tools have been subsequently extended to the verification of embedded software, real-time communication protocols, air traffic control, automotive control, bioengineering, process control, highway systems and manufacturing. Though many of the applications are still too complicated to be addressed in their full generality by existing hybrid tools, impressive progress has been recorded in all of these application areas.

The aim of this chapter is to highlight the diversity of hybrid phenomena that one encounters in physical and technological systems. In Section 2 a number of examples are presented to illustrate the types of issues that arise out of the discrete-continuous interaction and the types of applications that can be addressed using a hybrid approach. We also discuss the common themes that emerge in the study of these examples.

In Section 3 we present a formal mathematical framework, which we call *hybrid automaton*, in which all of these diverse phenomena can be modeled and analyzed. Then, in Section 4, we discuss how one can determine whether models developed in the hybrid automaton framework are reasonable representations of physical reality, or whether they contain fundamental flaws. Software tools for modeling hybrid systems are briefly discussed in Section 5.

2. Examples of Hybrid Systems

Hybrid control systems are a much richer class of systems than ordinary control systems. In a hybrid system there is an interaction between continuous and discrete dynamics. The continuous flow is in general influenced not only by the regular continuous control, but also by the discrete mode. Similarly, the discrete dynamics are affected by both discrete control actions and, indirectly, by the continuous flow. In addition to control inputs, there might be both continuous and discrete disturbances acting on the systems. Therefore, in its full generality, a hybrid control system can be a rather complicated object. In Section 3 we present a mathematical framework that allows one to model a class of hybrid phenomena. First, however, we informally introduce a number of examples, which are chosen to illustrate various characteristics of hybrid dynamics.

2.1. Water Tank System

Consider the two-tank system shown in Figure 1. For $i \in \{1, 2\}$, let x_i denote the volume of water in Tank i and $v_i > 0$ denote the constant flow of water out of Tank i. Let w denote the constant flow of water into the system, dedicated exclusively to either Tank 1 or Tank 2 at each time instant. The objective is to keep the water volumes above r_1 and r_2, respectively, assuming that the water volumes are above r_1 and r_2 initially. This is to be achieved by a controller that switches the inflows to Tank 1 whenever $x_1 \le r_1$ and to Tank 2 whenever $x_2 \le r_2$. The water tank systems can be represented by the hybrid system of Figure 1.

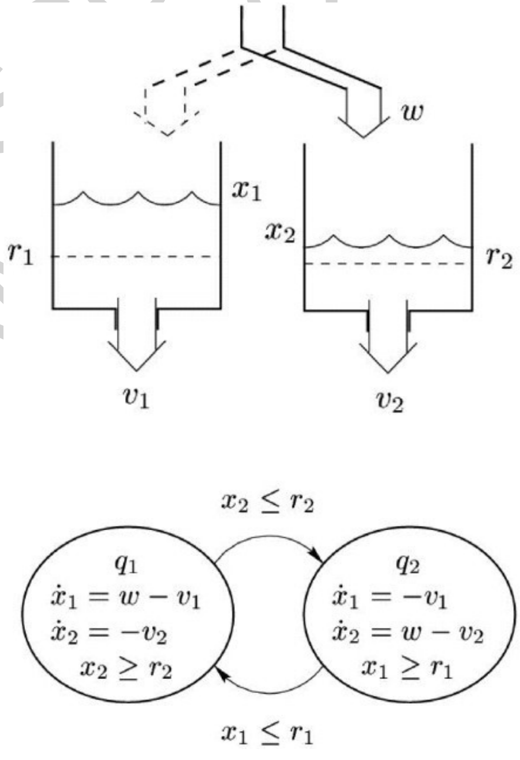

Figure 1: Water tank system and the corresponding hybrid system.

Suppose that at the initial time $x_1 \le r_1$ and $x_2 \le r_2$, and that the inflow if directed to Tank 1 (i.e., the discrete state q of the system is equal to q_1). Then the continuous state flows according to the differential equation in the q_1 state in Figure 1. When the condition $x_2 \le r_2$ (specified on the edge) is fulfilled, a discrete transition takes place. Subsequently, the state resumes flowing according to the q_2 state and so on. Such a trajectory having one continuous component, x, and one discrete component, q, is called an *execution* (sometimes a *run* or a *solution*) of the hybrid system. An execution of the hybrid system is shown in Figure 2.

If $\max(v_1, v_2) < w < v_1 + v_2$, physical intuition suggests that at least one of the water tanks will eventually drain. In the hybrid model this leads to an accumulation of jump instances. This behavior is known as the Zeno phenomenon and is further discussed in Section 4.

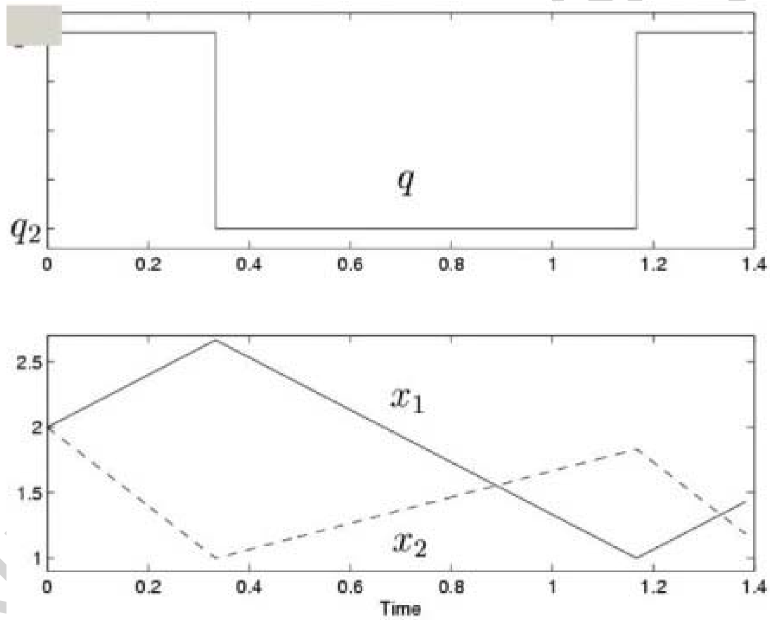

Figure 2: Example of an execution of the water tank hybrid system.

2.2. Bouncing Ball

A model for a bouncing ball can be represented as a simple hybrid system with a continuous state of dimension two $x = (x_1, x_2)$ and a single discrete state (Figure 3). x_1 denotes the vertical position of the ball and x_2 its velocity. The continuous motion of the ball, governed by Newton's laws of motion, is represented by the differential equation in the vertex of the graph, where g denotes the gravitational acceleration. As specified, the equation is only valid as long as $x_1 \ge 0$, i.e., as long as the ball is above the ground. The ball bounces when $x_1 = 0$ and $x_2 \le 0$, which is detailed by the left expression attached to the edge of the graph ($\wedge$ denotes the logical "and"). At each bounce, the ball loses a fraction of its energy. This is represented by the equation $x_2 := -cx_2$, where $c^2 \in [0,1]$ is the coefficient of restitution. (The notation ": =" should be interpreted as if x_2 is reset to the value $-cx_2$ at the transition. The *reset map* is formally defined in Section 3.)

Starting at a point (x_1, x_2) with $x_1 > 0$, the continuous state flows according to the vector field as long as the condition $x_1 \geq 0$ is fulfilled. When $x_1 = 0$ and $x_2 \leq 0$, a discrete transition takes place and the continuous state is reset to $x_2 := -cx_2$ (x_1 remains constant). Subsequently, the state resumes flowing according to the vector field and so on.

For this example, it is easy to see that for $c \in (0,1)$ there is an accumulation point for the times of the discrete jumps. In other words, the ball bounces infinitely many times in a finite time interval. The bouncing ball hence exhibit Zeno phenomenon, similar to the water tank system. Note however that the continuous state is constant at discrete transitions for the water tank system (the water volumes do not change during the switch of the inflow), while for the bouncing ball system the continuous state makes a jump.

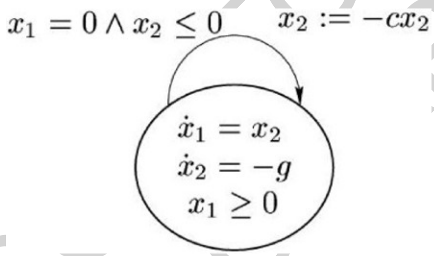

Figure 3: A hybrid system modeling bouncing ball.

2.3. Clegg Integrator

Many classical control strategies involve mode switching and other discontinuous control actions. Examples include anti-windup schemes, gain scheduling and sliding model control. One motivation for hybrid control models is to include all these strategies within a single mathematical framework. Here we describe a classical fix in process control, where the state of the integrator in the PID controller (see *Design Methods for Digital Controllers, Sample-Rate*) is reset whenever its input crosses zero. This so called Clegg integrator was invented by J.C. Clegg in 1958.

Let e be the input to the Clegg integrator and x the integrator state. The Clegg Integrator can be described by

$$\dot{x}(t) = e(t) \qquad \text{and} \qquad x(t+) = 0, \quad \text{if } e(t) = 0,$$

where the plus sign indicates that x is set to zero directly after e becomes zero. Figure 4 shows a hybrid model for the same set of equations. The hybrid system has the input e and the output x.

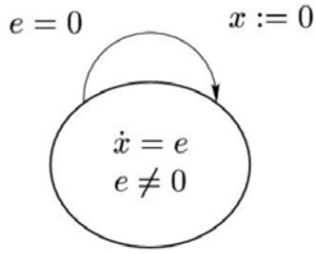

Figure 4: Hybrid system illustrating the Clegg Integrator.

The advantage of using a Clegg Integrator compared to an ordinary integrator is that it gives less phase lag, and thus in many applications improved stability margin. Using the describing function method (see *Describing Function Method*) it is easy to show that the Clegg Integrator gives 38 degrees phase lag, compared to 90 degrees of an ordinary integrator. A disadvantage with the Clegg Integrator is that it may induce oscillations.

2.4. Thermostat

Consider the control problem of maintaining the temperature of a room at some desired level (say 19 degrees Celsius). Assume that a thermostat is used as a controller, but that we do not have an exact model of how the thermostat functions. It is only known that the thermostat turns on the radiator when the temperature is between 16 and 18 degrees and it turns the radiator off when the temperature is between 20 and 22. This heating system can be modeled as the hybrid system in Figure 5, where x denotes the temperature and the two discrete states correspond to the radiator being off and on.

In this example, there is some uncertainty about when a transition takes place. We know that this will happen when the temperature is in the intervals [16,18] and [20,22], but not exactly when. Let us elaborate on how this ambiguity is captured by the hybrid system model. (A formal description is given in Section 3.) Note that there are three components associated with the discrete dynamics: (1) the *domains* $x \geq 16$ and $x \leq 22$, which constrain the values of the continuous state in the corresponding discrete state; (2) the *guard conditions* $x \leq 18$ *and* $x \geq 20$, which determine when a discrete transition is allowed to happen (is *enabled)*; and (3) the *reset map* $x \mapsto x$, which specifies the relation between the old and the new continuous state when a transition takes place (which in this example is equal to the identity map, but for the bouncing ball, for example, is $(x_1, x_2) \mapsto (x_1, -cx_2)$).

The interpretation is as follows: as long as the continuous state x belongs to a domain, continuous evolution *may* continue (the temperature may continue to increase/decrease according to the differential equation). When x enters a guard, a discrete jump *may* take place (the radiator may be switched on/off). For the thermostat system this means that if, for example, the state is $(q, x) =$ (no heating,19) then continuous evolution may continue. If the state is $(q, x) =$ (no heating,17) either continuous evolution can continue, or a discrete jump to state $(q, x) =$ (heating, 17) can take place. Finally, if the state is $(q, x) =$ (no heating, 16), a discrete jump must take place, because continuous evolution would lead x outside the domain.

The thermostat hybrid system is *non-deterministic*, in the sense that for a given initial condition it accepts a whole family of different executions. A formal definition of a hybrid systems and its evolution is given in Section 3, and determinism is discussed in Section 4.

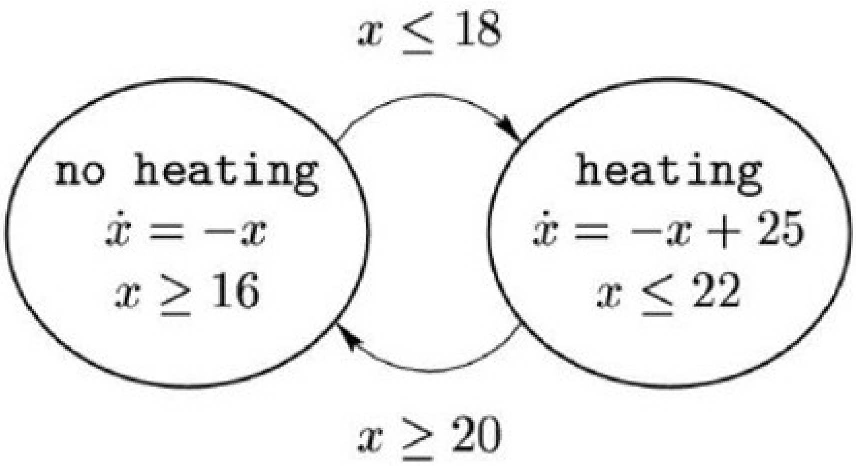

Figure 5: Hybrid system modeling a thermostat and the heating of a room.

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Biographical Sketches

Karl Henrik Johansson received the M.S. and Ph.D. degrees in electrical engineering, both from Lund University, Lund, Sweden, in 1992 and 1997, respectively. He held positions as Assistant Professor at Lund University (1997-1998) and as Visiting Research Fellow at the University of California, Berkeley (1998-2000). Currently, he is an Associate Professor in the Department of Signals, Sensors and Systems at the Royal Institute of Technology, Stockholm, Sweden. His research interests are in hybrid and switched systems, distributed embedded control, and applications in communication networks and automotive industry. Dr. Johansson received the Young Author Prize of the IFAC World Congress in 1996, the Peccei Award from IIASA, Austria, in 1993, and a Young Researcher Award from Scania, Sweden, in 1996.

John Lygeros completed a B.Eng. degree in Electrical Engineering in 1990 and an M.Sc. degree in Control in 1991, both at Imperial College, London. He then obtained a Ph.D. in 1996, from the Electrical Engineering and Computer Sciences department, University of California, Berkeley. He held a series of postdoctoral research appointments at the National Automated Highway Systems Consortium, M.I.T., and U.C. Berkeley. In parallel, he also worked as a part time Research Engineer at SRI International, and as a visiting professor at the Mathematics Department of the Universite de Bretagne Occidentale, France. Between 2000 and 2003 he was a University Lecturer at the Department of Engineering, University of Cambridge, Cambridge U.K. and a Fellow of Churchill College. Since March 2003 he has been an Assistant Professor at the Department of Electrical and Computer Engineering, University of Patras, Patras, Greece. His research interests include modeling, analysis and control of hierarchical, hybrid and stochastic systems, with applications to large-scale systems such as automated highways and air traffic management

Shankar Sastry received the Ph.D. degree from the University of California, Berkeley, in 1981. He became Chairman, Department of Electrical Engineering and Computer Sciences, the University of California, Berkeley, in January 2001. The previous year, he served as Director of the Information Technology Office at the Defense Advanced Research Programs Agency (DARPA). From 1996 to 1999, he was the Director of the Electronics Research Laboratory at the University of California, Berkeley, an organized research unit on the Berkeley campus conducting research in computer sciences and all aspects of electrical engineering. During his Directorship from 1996 to 99, the laboratory grew from 29 M to 50 M in volume of extra-mural funding. He is a Professor of Electrical Engineering and Computer Sciences and a Professor of Bioengineering. He was on the faculty of the Massachusetts Institute of Technology (MIT), Cambridge, as an Assistant Professor from 1980 to 1982, and Harvard University, Cambridge, MA, as a chaired Gordon Mc Kay Professor in 1994. He has held visiting appointments at the Australian National University, Canberra, the University of Rome, Italy, Scuola Normale and University of Pisa, Italy, the CNRS laboratory LAAS, Toulouse, France (poste rouge), Professor Invite at Institut National Polytechnique de Grenoble, France (CNRS laboratory VERIMAG), and as a Vinton Hayes Visiting Fellow at the Center for Intelligent Control Systems at MIT. His areas of research are embedded and autonomous software, computer vision, computation in novel substrates such as DNA, nonlinear and adaptive control, robotic telesurgery, control of hybrid systems, embedded systems, sensor networks, and biological motor control. Nonlinear Systems: Analysis, Stability and Control (New York: Spring-Verlag, 1999) is his latest book, and he has coauthored over 250 technical papers and six books. He has coedited Hybrid Control II, Hybrid Control IV, Hybrid Control V (New York: Springer-Verlag, 1995, 1997, and 1999, respectively). Hybrid Systems: Computation and Control (New York: Springer-Verlag, 1998), and Essays in Mathematical Robotics (NewYork: Springer-Verlag IMA Series). Books on Embedded Software and Structure from Motion in Computer Vision are in progress.

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