STABILIZATION THROUGH HYBRID CONTROL

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Contents

Summary

This chapter addresses the problem of controlling a dynamical process using a *hybrid controller*, i.e., a controller that combines continuous dynamics with discrete logic. Typically, the discrete logic is used to effectively switch between several continuous controls laws and is called a *supervisor*.

We review several tools that can be found in the literature to design this type of hybrid controllers and to analyze the resulting closed-loop system. We illustrate how these tools can be utilized through two case studies.

1. Introduction

The basic problem considered here is the control of complex systems for which traditional control methodologies based on a single continuous controller do not provide satisfactory performance. In *hybrid control*, one builds a bank of alternative candidate controllers and switches among them based on measurements collected online.

The switching is orchestrated by a specially designed logic that uses the measurements to decide which controller should be placed in the feedback loop at each instant of time. Figure 1 shows the basic architecture employed by hybrid control.

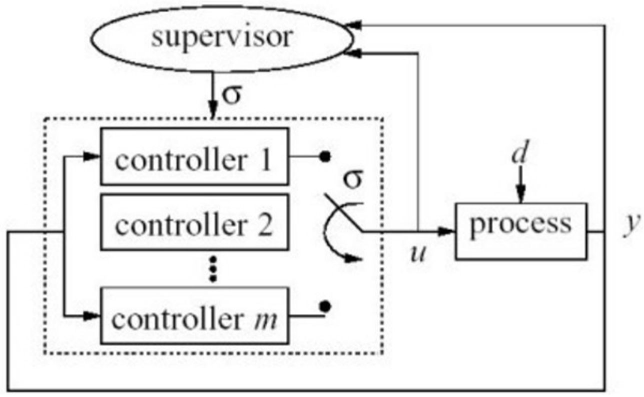

Figure 1. Hybrid control

In this figure u represents the control input, d an exogenous disturbance and/or measurement noise, and y the measured output. The dashed box is a conceptual representation of a switching controller. In practice, switching controllers are implemented differently. Suppose that we desire to switch among a family $\mathcal{C}$ of controllers parameterized by some variable $q \in \mathcal{Q}$. For example, we could have

$$\mathcal{C} := \{\dot{z}_q = F_q(z_q, y), u = G_q(z_q, y) : q \in \mathcal{Q}\},$$

where the set $\mathcal{Q}$ that parameterizes the functions $F_q(\cdot)$, $G_q(\cdot)$, $q \in \mathcal{Q}$ can be finite, infinite but countable, or not even countable (e.g., a ball in $\mathbb{R}^k$). Switching among the controllers in $\mathcal{C}$ can then be accomplished using the following *multi-controller*:

$$\dot{x}_C = F_\sigma(x_C, y), \qquad u = G_\sigma(x_C, y), \tag{1}$$

where $\sigma : [0, \infty) \to \mathcal{Q}$ is a piecewise constant signal—called the *switching signal*—that effectively determines which controller is in the loop at each instant of time. The points of discontinuity of σ correspond to a change in candidate controller and are therefore called *switching times*. The multi-controller in (1) is far more efficient than the conceptual structure in Figure 1 as its dimension is independent of the number of candidate controllers. Moreover, if some of the controllers in Figure 1 were unstable, their interval states could become unbounded if they were left out of the feedback loop. These issues are further discussed in [Morse, 1995]. In this chapter, we use a continuous-time multi-controller such as (1) to keep the exposition concrete. However, the concepts presented generalize to other types of candidate control laws, such as discrete-time [Borelli et al., 1998] or hybrid controllers [Hespanha et al., 1999].

The top element in Figure 1 is the logic that controls the switch, or more precisely, that generates the switching signal in (1). This logic is called the *supervisor* and its purpose is to monitor the signals that can be measured (in this case u and y) and decide, at each instant of time, which candidate controller should be put in the feedback loop with the process. In hybrid control, the supervisor combines continuous dynamics with discrete logic and is therefore a *hybrid system*. A typical hybrid supervisor can be defined by an ordinary differential equation coupled with a recursive equation such as

$$\dot{\varphi} = \mathbf{\Psi}_{\sigma}(\varphi, u, y), \qquad \mathbf{\Gamma}_{\sigma} = (\varphi, \sigma^{-}), \tag{2}$$

where $\{\mathbf{\Psi}_q(\cdot) : q \in \mathcal{Q}\}$ is a family of vector fields, and $\mathbf{\Gamma}(\cdot)$ a discrete transition function. A pair of signals (φ, σ) is called a *solution* to (2) if σ is piecewise constant taking values in $\mathcal{Q}$, φ is a solution in the sense of Carathéodory to the time-varying differential equation

$$\dot{\varphi} = \mathbf{\Psi}_{\sigma(t)}(\varphi, u(t), y(t)), \quad t > 0$$

and, for every $t > 0$,

$$\sigma(t) = \mathbf{\Gamma}(\varphi(t), \sigma^{-}(t)).$$

The signal φ is called the *continuous state* of the supervisor and σ its *discrete state*. We assume here that all signals of interest are continuous from above, and, given a piecewise continuous signal σ, we denote by σ^{-} the signal defined by $\sigma^{-}(t) = \lim_{\tau \uparrow t} \sigma(\tau)$, $t > 0$. More general models for hybrid systems and more sophisticated notions of solution can be found in *Modeling of Hybrid Systems* and in the works of [Tavernini, 1987; Morse *et al.*, 1992; Back *et al.*, 1993; Nerode and Kohn, 1993; Antsaklis *et al.*, 1993; Brockett, 1993; Branicky *et al.*, 1994; Lygeros *et al.*, 1999; Zhang *et al.*, 2000].

Hybrid control systems, like the one depicted in Figure 1, are used in many situations, such as:

1. When the performance requirements for the closed-loop system change over time. In this case, the supervisor is responsible for placing in the feedback loop the controller that is most suitable for the current needs.

2. When there is large uncertainty in the process to be controlled and offline identification is not possible or desirable. Here, the supervisor should place in the feedback loop the controller that is more likely to stabilize the actual process and provide adequate performance. This type of hybrid control can be viewed as a form of adaptive control, where switching replaces the more traditional continuous tuning. This type of hybrid control is considered in the case study in Section 4.2.

3. When the nature of the process requires hybrid stabilization. This can occur because there are fundamental limitations on the type of controllers that are able to stabilize the process or because the actuation or sensing mechanisms naturally result in switching control laws. Examples of the former are nonholonomic systems (cf., *Control of NonlinearSystems* and Brockett, 1983) and of the later are systems for which actuation is achieved through on-off valves or switches, or when the sensors used for feedback have a limited range of operation (cf. case study in Section 4.1).

The reader is referred to the works of [Morse, 1995; Hespanha, 1998; Eker and Malmborg, 1999; Lemmon *et al.*, 1999; Liberzon and Morse, 1999; DeCarlo *et al.*, 2000] and references therein for additional examples.

The interconnection of a process modeled by an ordinary differential equation, the multi-controller (1), and the hybrid supervisor (2), results in a hybrid system of the form

$$\dot{x} = A_\sigma(x, d), \qquad \sigma = \Phi(x, \sigma^-), \tag{3}$$

where the continuous state x takes value in $\mathbb{R}^n$, the discrete state σ is the switching signal that takes values in $\mathcal{Q}$, and d the process' exogenous disturbance. The analysis of this type of systems has been actively pursued in the last years. In particular, considerable research has been carried out to answer: reachability questions such has

Given two disjoint sets $\mathcal{S}, \mathcal{R} \subset \mathbb{R}^n \times \mathcal{Q}$, if the state (x, σ) of (3) starts inside $\mathcal{S}$, will it ever enter $\mathcal{R}$?

liveness questions such has

Given two discrete states $q_1, q_2 \in \mathcal{Q}$, will there be an infinite number of switching times at which σ switches from q_1 to q_2?

or stability questions such as

Will the solution to (3) exist globally and, if so, will the continuous state x remain uniformly bounded and the output y converge to some set-point r as $t \to \infty$?

In this chapter we are mostly interested in stability questions such as the last one. Note that with hybrid systems like (3), global existence of solution may fail either because the continuous state x becomes unbounded in finite time—often called *finite escape time*—or because the discrete state σ exhibits an infinite number of switches in finite time—often called *chattering* or the *Zeno phenomenon* (cf. *Modeling of Hybrid Systems*, *Well-posedness of Hybrid Systems* and Johansson *et al.*, 1999).

There is no systematic procedure to study the stability of a generic hybrid system. However, the arguments used to prove the stability of hybrid systems usually consist of consecutively applying results of the type

PD: *Assuming that x belongs to a family $\mathcal{X}_k$ of signals taking values in $\mathbb{R}^n$, then the discrete state σ belong to the family $\mathcal{S}_k$ of switching signals.*

PC: *Assuming that σ belongs to a family $\mathcal{S}_k$ of switching signals, then the continuous state x belongs to the family $\mathcal{X}_{k+1}$ of signals taking values in $\mathbb{R}^n$.*

until one concludes that x belongs to some family of uniformly bounded signals $\mathcal{X}_n$ with the desired asymptotic properties. A result of the PD type corresponds to a property of the discrete-logic

$$\sigma = \Phi(x, \sigma^-), \qquad t \geq 0, \tag{4}$$

whereas a result of the PC type corresponds to a property of the continuous-time switched system

$$\dot{x} = A_\sigma(x, d).$$

In the following sections we present several results of these types that are available in the literature. Section 2 focus on PC results, whereas Section 3 concentrates on PD results. Many of these lead directly to hybrid controller design methodologies. This is illustrated in Section 4 through two case studies.

For lack of space, we do not pursue analysis techniques based on *impact* or *Poicaré return maps*. The basic idea behind impact maps is to "sample" the continuous state at switching times and then analyze its evolution as if one was dealing with a discrete-time system.

The main difficulty with this type of approach is that, because the sampling is not uniform over time, even for simple continuous dynamics (e.g., linear or affine), the "sampled" system may be very nonlinear and it may even be difficult to write it explicitly.

However, this type of technique was used successfully, e.g., by [Grizzle *et al.*, 2001] to analyze bipedal walking robots and by [Gonçalves *et al.*, 2001] to analyze relay feedback systems.

2. Switched Systems

In this section we study the properties of a continuous-time switched system of the form

$$\dot{x} = A_\sigma(x, d), \qquad x \in \mathbb{R}^n,\ d \in \mathbb{R}^k, \tag{5}$$

where the family of vector fields $\{A_q(\cdot) : q \in \mathcal{Q}\}$ is given and the switching signal $\sigma : [0, \infty) \to \mathcal{Q}$ is known to belong to some set $\mathcal{S}$ of piecewise-constant signals.

We recall that $\mathcal{K}$ denotes the set of all continuous functions α : [0,∞) → [0,∞) that are zero at zero, strictly increasing, and continuous; $\mathcal{K}_\infty$ the subset of $\mathcal{K}$ consisting of those functions that are unbounded; and $\mathcal{KL}$ the set of continuous functions β : [0,∞)×[0,∞) → [0,∞) which, for every fixed value of the second argument, are of class $\mathcal{K}$ when regarded as functions of the first argument, and that have $\lim_{\tau\to\infty}$ $ß(s, t) = 0$ for every fixed $s \geq 0$. Given a vector $x \in \mathbb{R}^n$ we denote by $\|x\|$ the Euclidean norm of *x*.

We say that (5) is *uniformly asymptotically stable* over $\mathcal{S}$ if there exists a function β of class $\mathcal{KL}$ such that, for every $\sigma \in \mathcal{S}$,

$$\|x(t)\| \leq \beta(\|x(\tau)\|, t-\tau), \qquad \forall t \geq \tau \geq 0, \tag{6}$$

along solutions to (5) for which $d(t)=0$, $t \geq 0$. When $\beta(s,t)$ is of the form $ce^{-\lambda t}s$ for some $c, \lambda > 0$ we say that (5) is *uniformly exponentially stable over* $\mathcal{S}$. In this case we can emphasize the rate of decay in the above bound by adding that (5) has *stability margin* λ. Local versions of these definitions can be obtained by restricting $x(\tau)$ in (6) to belong to an open neighborhood of the origin.

For exogenous inputs d that are not necessarily zero, we say that (5) is uniformly *input-to-state stable over* $\mathcal{S}$ if there exists a function α of class $\mathcal{K}$ and a function β of class $\mathcal{KL}$ such that, for every $\sigma \in \mathcal{S}$,

$$\|x(t)\| \leq \beta(\|x(\tau)\|, t-\tau) + \sup_{s \in [\tau, t)} \alpha(\|d(t)\|), \qquad \forall t \geq \tau \geq 0, \tag{7}$$

along solutions to (5). Replacing the $\sup_{s \in (t-\tau)}$ in (7) by the integral $\int_\tau^t \cdot\, ds$ over the same interval, we obtain the definition of *uniform integral-input-to-state stability over* $\mathcal{S}$.

When all the vector fields $A_q(\cdot)$, $q \in \mathcal{Q}$ are linear we say that (5) is a linear *switched system*. In case the set of matrices that represent these maps in some basis of $\mathbb{R}^n$ is compact, (5) is called a *compact linear switched system*. Compactness is automatically guaranteed whenever $\mathcal{Q}$ is finite.

For compact linear systems, one can use fairly standard results to prove that uniform asymptotic stability is equivalent to uniform exponential stability (cf., e.g., the work of Molchanov and Pyatnitskiy, 1989, for details).

Similar to what happens for unswitched linear systems, uniform exponential stability of a compact linear switched system over $\mathcal{S}$ implies uniform input-to-state and integral-input-to-state stability over the same set $\mathcal{S}$.

In fact, uniform exponential stability over $\mathcal{S}$, actually implies that several induced norms of (5) are uniformly bounded over $\mathcal{S}$. We define some of these norms next:

Given a positive constant λ, we say that (5) has *input-to-state* $e^{\lambda t}$*-weighted,* L_∞*-induced norm uniformly bounded over* $\mathcal{S}$ if there exist finite constants g, g_0 such that, for every piecewise continuous input d and every $\sigma \in \mathcal{S}$,

$$e^{\lambda t}\|x(t)\| \le g_0\, e^{\lambda \tau}\|x(\tau)\| + g \sup_{[\tau,t)} e^{\lambda s}\|d(s)\|, \qquad t \ge \tau \ge 0. \tag{8}$$

In general, this is stronger than uniform input-to-state stability because (8) implies (7) with $\beta(s,t) = g_0 e^{-\lambda t} s$ and $\alpha(s) = gs$, $t, s \ge 0$. When (8) is replaced by

$$e^{\lambda t}\|x(t)\| \le g_0\, e^{\lambda \tau}\|x(\tau)\| + g\left(\int_0^t e^{2\lambda s}\|d(s)\|^2\, ds\right)^{\frac{1}{2}}, \quad t \ge \tau \ge 0, \tag{9}$$

we say that (5) has input-to-state $e^{\lambda t}$-weighted, *L_2-to-L_∞-induced norm uniformly bounded over $\mathcal{S}$*. In general, this is stronger than uniform integral-input-to-state stability because (9) implies that (7) holds with $\sup_{s\in(t-\tau)}$ replaced by $\int_\tau^t \cdot\, ds$, $\beta(s,t) = g_0 e^{-\lambda t} s$, and $\alpha(s) = gs$, $t, s \ge 0$. To verify that this is true one needs to use the fact that $(\int_a^b x^2)^{\frac{1}{2}} \le \int_a^b |x|$ for every signal x for which the integrals exist. Finally, if (8) is replaced by

$$\left(\int_0^t e^{2\lambda\tau}\|x(\tau)\|^2\right)^{\frac{1}{2}} \le g_0\|x(0)\| + g\left(\int_0^t e^{2\lambda\tau}\|d(\tau)\|^2\right)^{\frac{1}{2}}, \qquad t \ge 0, \tag{10}$$

we say that (5) has *input-to-state $e^{\lambda t}$-weighted, L_2-induced norm uniformly bounded over $\mathcal{S}$* It is straightforward to show (cf., e.g., Hespanha and Morse, 1999b) that the following holds.

Lemma 1. *Suppose that (5) is a compact linear switched system. Given a family $\mathcal{S}$ of piecewise constant switching signals, if (5) is uniformly exponentially stable over $\mathcal{S}$, with stability margin λ_0, then, for every $\lambda \in [0, \lambda_0)$, (5) has input-to-state $e^{\lambda t}$-weighted, L_∞-induced norm uniformly bounded over $\mathcal{S}$. Similarly for the L_2 and L_2-to-L_∞ induced norms.*

The computation of L_2-induced norms for switched linear systems was studied by [Hespanha, 2002], which showed that even for very slow switching the induced norm of a switched system can be strictly larger than the norms of the systems being switched.

In fact, the induced norm of a switched system is realization dependent and cannot be determined just from the transfer functions of the systems being switched.

We proceed to analyze the uniform stability of switched systems over several classes of switching signal.

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