MODELING AND CONTROL OF COMPLEX RIVER AND WATER RESERVOIR SYSTEMS

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Contents

Summary

This article is an introductory treatment of modeling and control of flows and/or level in catchment areas and rivers. The model concepts are based on the Saint-Venant-Equations and linear/ nonlinear difference equations. The control of flow and level uses decentralized and hierarchial control structures.

1. Introduction

Modeling and control of flow and / or level in catchment areas, rivers and channels has been the focus of a tremendous amount of activity in the past few years. The prime objectives of a water management system are flood prediction, ensurance of navigation safety, production of hydroelectricity and the maintenance of water supply for the household, industry, agriculture and the environment.

Water systems are characterized by a complex, highly non-linear dynamic, uncertain and locally distributed hydraulic behaviour, which is mainly determined by meteorological conditions (e.g., rain, snow, temperature), hydro-geological conditions (e.g., geometrical, surface parameters) and the control decisions made by a high order controller or an expert.

A widely accepted approach in the computer aided design and realization of decision systems (stand-alone or as an operator-guide) is to initially develop suitable models to describe the non-linear process dynamics. The structure and the parameters of the models are chosen or estimated respectively, depending on the component of the water system to be modeled.

Modeling concepts for monitoring, control and prediction are based on theoretical laws (mathematical equations) and their parameters are estimated from the measured data on the inputs and the outputs of the process (see *General Models of Dynamic Systems, Identification of Linear Systems in Time Domain, and Identification of Nonlinear Systems*). In general, the so called “hydrodynamic control models” (HDC-models) are based on the Saint-Venant-Equations.

A model aided computer simulation realizes the optimal design of the decision systems. In the real water system, the parameters of the decision system are tuned through adaptation by learning algorithms.

2. Models of water plants

2.1. Catchment area models

The catchment area models describe the behaviour of hydrological processes, such as snow melting process and/or rainfall – runoff process.

2.1.1. Snow melting process models (snow-melt-models)

The basic structure of the model of the snow melting process is shown in Figure 1.

The sum of the percolating precipitate and the effective amount of water from the molten snow is the input variable (groundwater $Q_G(kT)$) to the rainfall – runoff process model (see Figure 2). All the models presented in this paper are a discrete time representation of the process in consideration (i.e., a discrete time representation implies measurement or sampling of the process variable at discrete time intervals, kT, where T is the sampling period and k is an integer). A detailed description of the data sampling aspects of the measurements is given in Section 2.2.1.

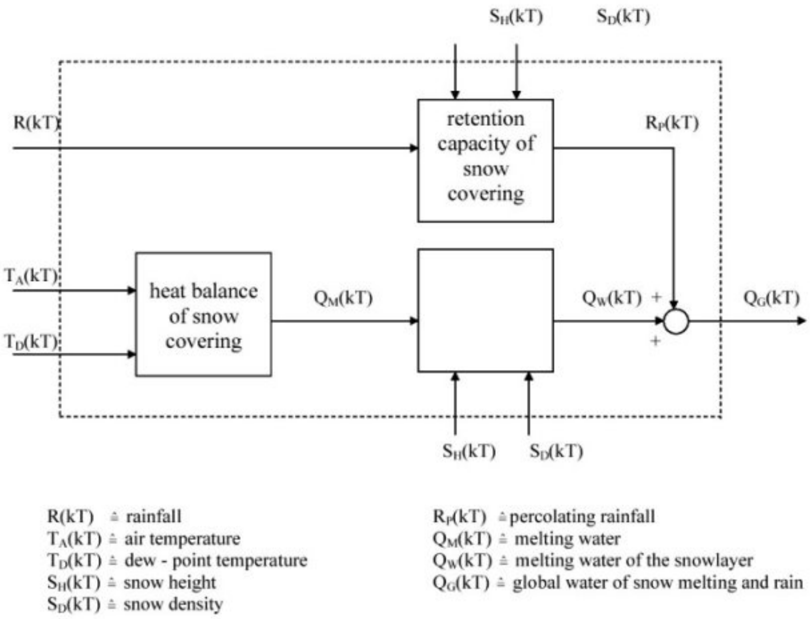

Figure 1. Structure of the snow melting process model

2.1.2. Rainfall – runoff process models (Rainfall-runoff-models)

There are many models that can be used to describe the rainfall-runoff process, but for the purposes of control and prediction, the model structure (see Figure 2) has proven to be very effective.

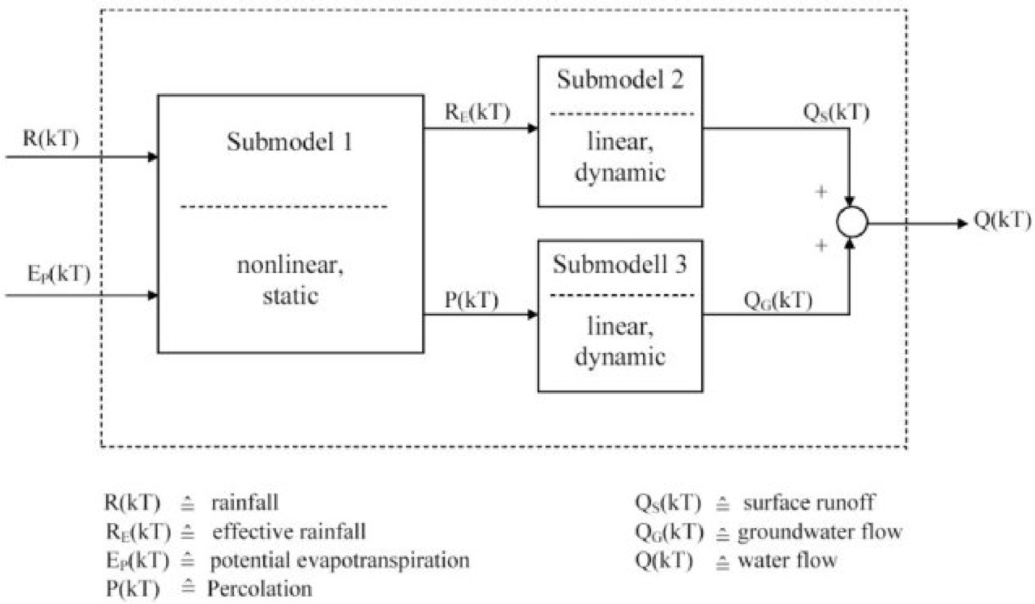

Figure 2. Structure of the rainfall-runoff process model

Lorent and Gevers' approach describes a simplified model of the catchment area. It comprises a nonlinear static sub-model of the soil moisture (to calculate the effective rainfall and seepage) and two linear dynamic sub-models to convert the effective rainfall into surface discharge as well as the seepage into base discharges. The total discharge at the gauge of the catchment area is then the result of the sum of the surface discharge and the base discharges.

Another simple, but sufficing modeling concept for control purposes is given below. It describes the behaviour of the effective rainfall and the resulting discharge of the catchment area.

$$Q[(k+1)T] = -\sum_{i=1}^{n} a_i Q[(k-i)T] + \sum_{j=0}^{m} b_j R_e^*[(k-j)T] \qquad (1)$$

where Q is the flow in m^3/s, R_e is the effective rainfall in mm/h, $R_e^* \cong c_0 R_e + c_1 R_e^2 + \dots$ is the non-linear rainfall in mm/h; a, b are weighting factors.

The model structure described by Equation1 is characterized by a robust and adaptive behaviour, which makes its application more flexible.

2.2. River models

The behavior of a river between a given input and output location can easily be represented by sub-models describing typical hydraulic characteristics (e.g., water flow velocities, increases or decreases in the water level, the rate of overflow and the behaviour in retention rooms) of the river.

2.2.1. Waterflow – models

A widely accepted approach in the description of the behavior of the water flow rate of a river between a given input and output location is to apply non-linear difference equations with time delay T_t (see Figure 3). These models can be expressed as

$$Q_0[(k)T] = -\sum_{i=1}^{n} a_i Q_0[(k-i)T] + \sum_{j=0}^{m} b_j Q_I^*[(k-j-d)T] \qquad (2)$$

where Q_I is the input flow in m^3/s; $Q_I^* = c_0 Q_I + c_1 Q_I^2 + \dots$ is the non-linear input flow in m^3/s; Q_0 is the output flow in m^3/s and $d = \frac{T_t}{T}$ (T_t is the time delay of the system and T is the sampling period).

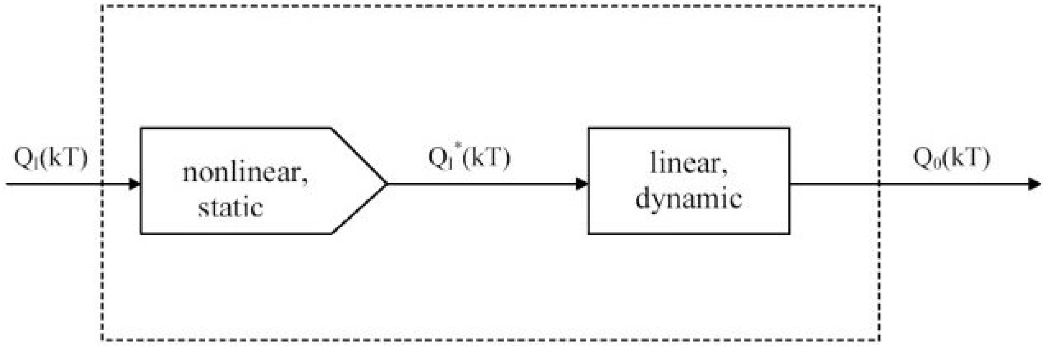

Figure 3. Structure of the water flow models

A second model structure was developed based on the classical Saint-Venant-Equations for one-dimensional channel flow. The continuity equation (detailed description of the Equations 3-9 are given in 14, 15) can be expressed as

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \qquad (3)$$

where Q is the flow; x is the distance; A is the cross-sectional area; t is the time.

The equation of momentum is given by:

$$\frac{\partial H}{\partial x}+\frac{1}{g}\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)+I_R-I_s=0 \qquad (4)$$

where H is the depth of water: v is the flow speed; I_R is the coefficient of friction; I_S is the gradient of the riverbed.

Although most real-life processes are inherently continuous-time processes, for ease of computation in modeling or other applications, a discrete time representation of the process is considered. The numerical solutions of the above equations are preferred for hydro-geological aspects, whereas on the other hand, to solve control problems, a segmentation in small river sections will result in excessive estimation and tuning effort of the parameters. Hence, the computational effort for online purposes will be unrealistically high. To solve control problems, it is sufficient to approximate some parts of the equation of momentum (Equation 4) with the Strickler-Equation and the Taylor series expansion, leading to

$$\frac{\partial Q}{\partial x}+B\frac{\partial A}{\partial t}=0$$

$$c_1\frac{\partial H}{\partial x}+c_2\frac{\partial Q}{\partial x}+c_3\frac{\partial Q}{\partial t}=-a_1Q+a_2H-a_4Q^2+a_5QH-a_6H^2+\ldots \qquad (5)$$

where B is the width of the river. Hence, measurements of the river cross-section representing distinctive sections of the river, must be available in a priori. In reality, the coefficients c_2, c_3, a_4, etc. are negligibly small ($\approx 10^{-4}$). Hence,

$$c_1 \frac{\partial H}{\partial x} = -a_1 Q + a_2 H \tag{6}$$

or

$$Q = c_1^* H - c_2^* \frac{\partial H}{\partial x}$$

In other terms, Equation 6 describes the global flow behavior, which can be expressed as follows:

$$Q = Q_{stationary} + Q_{non-stationary} \tag{7}$$

where $Q_{stationary} = k_1 H$ and $Q_{non-stationary} = -k_2 B \frac{\partial H}{\partial x}$.

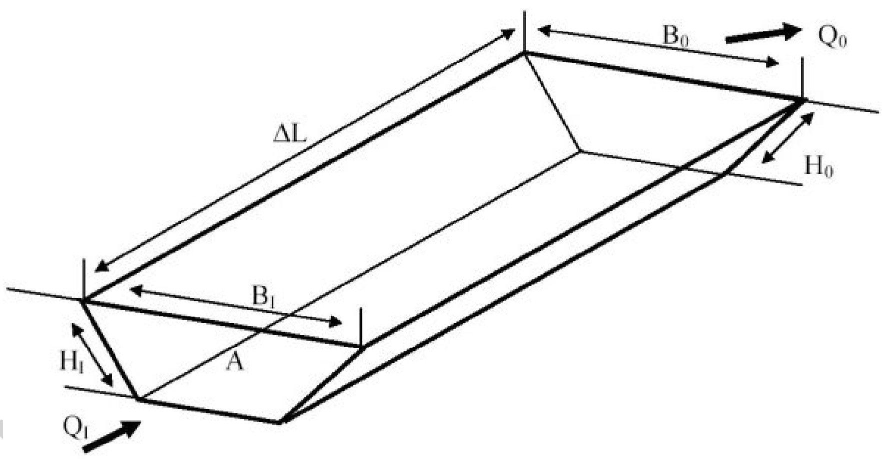

Figure 4. The Global Flow Behaviour

Figure 5 shows how the flow Q varies with the water level H.

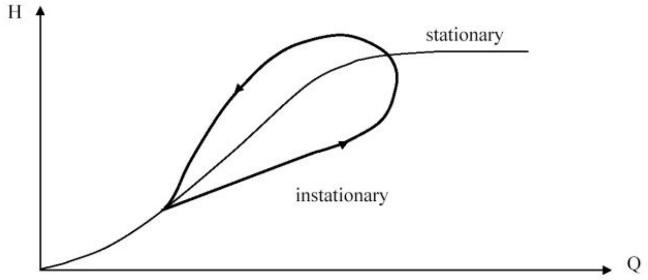

Figure 5. Characteristic curves of stationary / non-stationary behavior of the river

Differentiating Equation 6 with respect to time (t) and assuming that the river width B is constant results in Equation 8

$$\frac{\partial Q}{\partial t} = -\frac{c_1^*}{B}\frac{\partial Q}{\partial x} - c_2^* \frac{\partial^2 H}{\partial x \partial t} \qquad (8)$$

Although the width of a natural river (not man made) is never constant, this assumption is justified, because the parameters used in the model are estimated from measured flow and water level data of the real system by using least squares method (LMS). In this way, one adapts the model to the behavior of the real river.

In discrete time form (i.e., $H(x,t) \Rightarrow H[m\Delta l, kT]$ and $Q(x,t) \Rightarrow Q[m\Delta l, kT]$ where Δl is the length of the river section; T is the sampling period, Equation 2 can be expressed as follows:

$$\begin{aligned} Q_0[m_0,k] &= a_1 Q_0[m_0,(k-1)] + a_2 Q_0[m_0,(k-2)] + \dots \\ &\quad + b_1 Q_I[m_I,(k-k_1)] + b_2 Q_I[m_I,(k-1)] + \dots \\ &\quad + c[H_0(m_0,(k-1)) - H_I(m_I,(k-1)) - H_0(m_0,(k-2)) + H_I,[m_I,(k-2)]] \end{aligned} \qquad (9)$$

where $m_0\Delta l$ is the start location of the river section; $m_I\Delta l$ is the end location of the river section.

The structure of the model is illustrated in Figure 6.

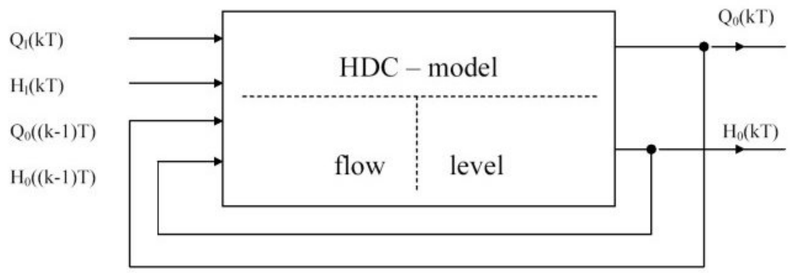

Figure 6. Structure of hydrodynamic control model (HDC-model)

2.2.2. River water level models

Figure 7. Structure of the water level model (stationary behavior)

The water level of a given location on the river course is estimated from the linear/non-linear characteristic curves of the river flow and level (Figure 7). The main problem hereby is the estimation of the characteristic curves.

The Saint-Venant-Equation (Equation 4) is used to determine the water level at any location of the river. Hence, the change in the water level can be expressed as

$$\partial H = I_s \partial L - I_R \partial L - \frac{v}{g}\partial v - \frac{1}{g}\frac{\partial v}{\partial t}\partial L \qquad (10)$$

where $v = \frac{Q}{A}$ is the flow velocity; Q is the water flow. A is the cross-sectional area of the river. In discrete time form the water level at location L_1 (see Figure 8) of the river section can be expressed as

$$H_1(kT) = H_2(kT) + H_{02} - H_{01} - \Delta H_2(kT)$$

Where

$$\Delta H(kT) = -c_1 v_m(kT)^2 \Delta L - \frac{v_m(kT)}{g}\left[v_2(kT) - v_1(kT)\right]$$

and

$$v_m(kT) = \frac{v_1(kT) + v_2(kT)}{2} \qquad (11)$$

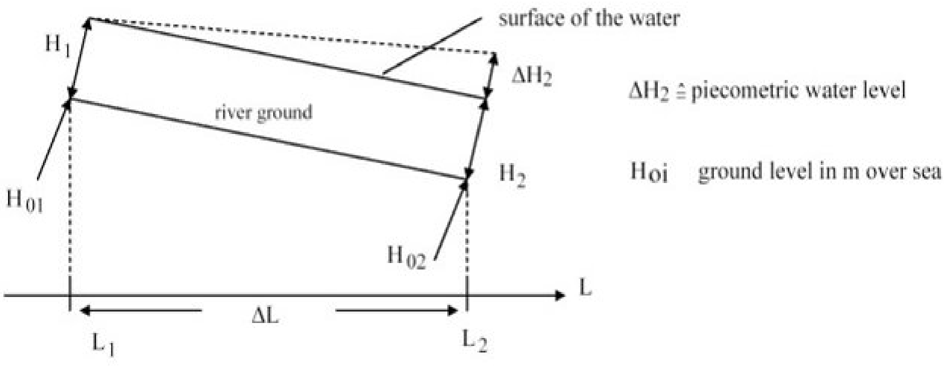

Figure 8: Discrete time from the water level at location L_1

Accordingly, the water level at any location of a river or dam can be computed using Equation 11, the so-called “Bresse-differential-equation”. It is assumed that *a priori* knowledge of the cross-sectional area of the river or dam at that definite location is available.

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Biographical Sketch

Jürgen Wernstedt was born in Hall/Saale, Germany, in 1940. Since 1985 he has been Professor for System Analysis, Ilmenau Technical University, Department of Informatic ant Automatic Control, Germany.

He teaches courses in automatic control, modeling, system analysis, fuzzy control, decision systems. The research areas are modeling, knowledge based system, fuzzy control. He has been responsible for 32 Ph. D. Theses, published more than 120 technical papers in journals and technical meetings and is author / co-author of two internationally published books to modeling and fuzzy control.

Since 1995 he has been the leader of the Application Center System Technology Ilmenau of the Fraunhofer Society. The activities are the planning and management of large-scale energy and water-supply system.